

Management of a high mix production system with interdependent demands:  
global and decomposed optimization approaches for inventory control

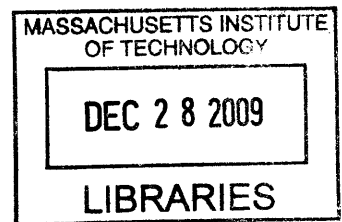
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# Abstract

In this work the management of a production system with a high mixture of products, interdependent demands and optional components is analyzed. An approach based on reorder point policy is proposed for both raw parts and finished goods inventory control. In the latter case, the solution of an optimization problem determines whether each product should be held in inventory and if so which safety factor  $z$  should be used. The choice of  $z$ , and as a consequence of the reorder level  $R$ , takes into consideration the demands interdependence, the customer's availability to buy if a certain waiting time is quoted to them and the fact that in a certain type of orders the optional components are required to ship. Global and decomposed optimization approaches are presented and compared. The decomposed approach is shown to achieve performances very close to the ones of the global optimization with much easier computations. By using the policy based on the decomposed optimization, it is possible to reduce simultaneously the value of the inventory and the expected number of lost sales as compared to a simple reorder point policy or to the policy currently in use at the company. A reduction of 35% of inventory and of almost 10 times of the average value of lost sales is expected if the company substitutes the current policy with the proposed one.

## **List of Notation**

$I$  : Inventory

$S$  : Sales

$\varsigma$  : Systems orders

$\mathcal{O}$  : Otc orders

$Q$  : Reorder Quantity

$R$  : Reorder Level

$h_i$  : holding cost of item  $i$

$p_i$  : Unit Profit of item  $i$

$P_i$  : Total Profit of item  $i$  during a period

$V_i$  : Virtual Profit of item  $i$

$c_i$  : Unit Cost of item  $i$

$L$  : Lead Time

$f_L$  : Pdf of demand over lead time

$\alpha$  : Type I Service Level

$\beta$  : Type II Service Level

$U$  : Number of Orders

$T$  : Number of Items

$\Omega_j$  : Set of items part of order  $j$

$\omega_j = \dim(\Omega_j)$

$\varepsilon$  : opportunity cost

$\mu(x)$  : Mean of random variable  $x$

$\sigma(x)$  : Standard deviation of random var.  $x$

$\varphi_{\eta,\sigma}(x)$  : Normal pdf (mean  $\mu$ , dev.  $\sigma$ )

$\varphi(x)$  : Standard Normal pdf

$\Phi_{\eta,\sigma}(x)$  : Normal cdf (mean  $\mu$ , dev.  $\sigma$ )

$\Phi(x)$  : Standard Normal cdf

$z$  : Safety factor

$\delta$  : Demand shift factor

$\rho_j$  : Profit made by order  $j$

$n_{i,j}$  : number of times item  $i$  is in order  $j$

$\gamma(z)$  : Function defined in g.3

$\zeta(z)$  : Function defined in g.8

$\xi(z)$  : Function defined in h.4

$x \in (a,b) = \{x \in \mathbb{R} \mid a < x < b\}$

$x \in [a,b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$

$v$  : number of items considered for inventory control

# **1 Introduction**

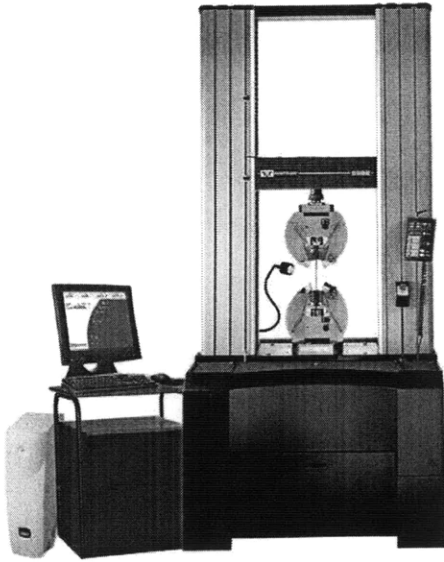
## **1.1 Instron Corporation as a Research Environment**

Founded in 1946, Instron® is the recognized worldwide market leader in the materials testing industry, holding more than 50% of the market share. The company has various products with all of them sharing production lines. The products cover the following areas of testing: fatigue, tension, compression, flexure, hardness, impact, torsion, spring, test analysis, structural and custom testing. Within each of these categories, many combinations of machines and accessories (hereafter called systems) are possible according to the customer's requirement. That is, all the testing equipment can be customized by the customer. Thus, even the same requirement of two customers may not result in the same order.

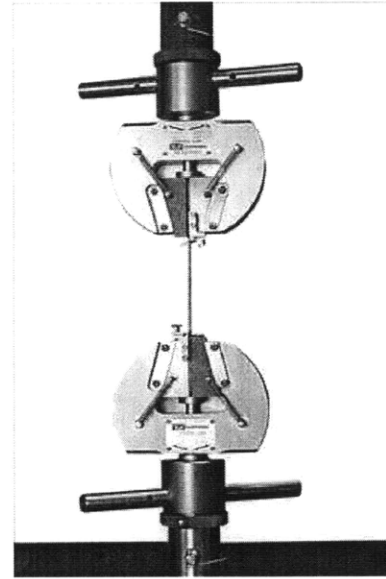
Such market behavior forces Instron to keep multiple product lines which further translate into a high inventory, low output factory floor. Thus, Instron serves more than a 'job-shop' volume but at the same time maintains a flexible manufacturing facility to produce highly customized products in minimum time. This issue is clearly visible in the accessories business of the electromechanical division. This area of the production line has the maximum variability and hence is an effective bottleneck. It is well known in the inventory management industry that rather than high demand, it is the variability that is the real reason behind the difficulty in managing service levels [1]. Thus, it is very important to make this area ready for such variability. This can only happen if the right mix of accessories is available at the right time, in the right quantities and at the right place.

Variability is not the only concern while dealing with the inventory in the EM accessories business. There are other intricacies involved which make the problem more challenging. For example, not only can the finished goods be sold as part of a system but they can also be sold as individual after sales parts (hereafter called as OTC - Over The Counter

products). Secondly, each system has to wait until all the items in it are available and only then it can be shipped. Thus, in the case of a system order, there is dependence of demand between these items, and they cannot be viewed as separate entities.

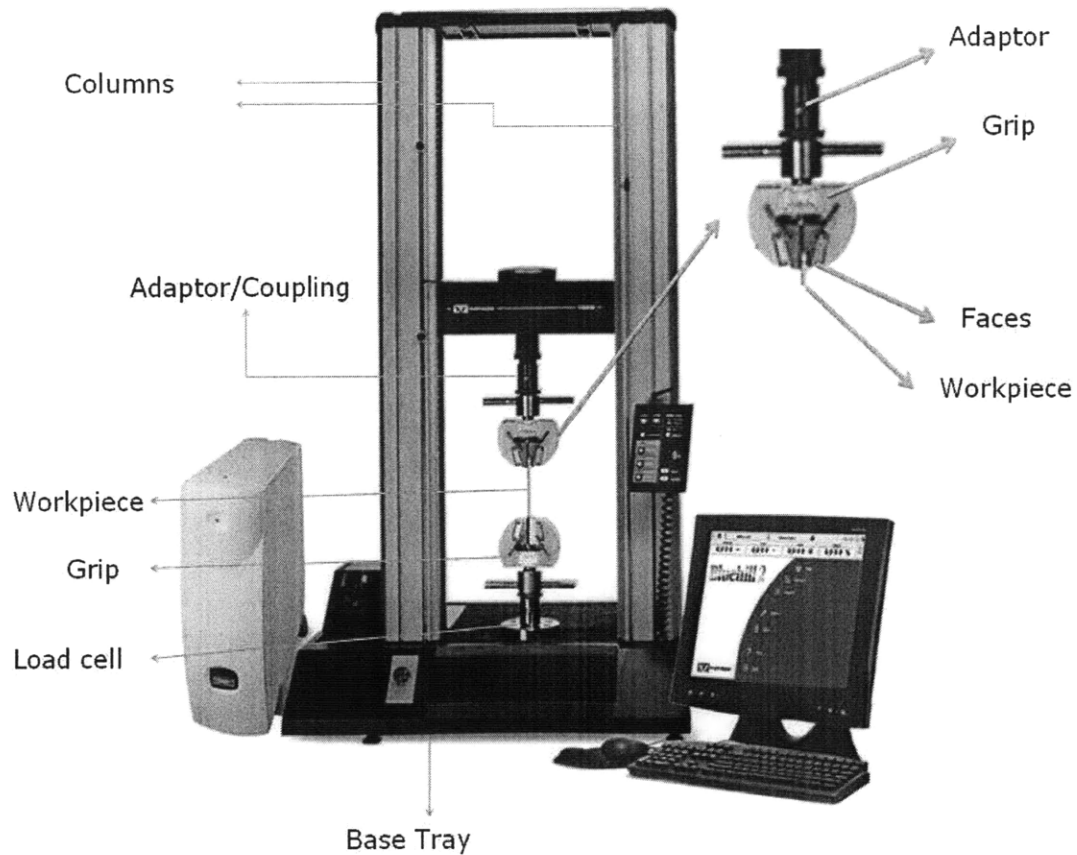


**Figure 1.1 - 5800 Series System**



**Figure 1.2 – A Wedge Action Grip**

The figures above show some products offered by the Electro-Mechanical business. Figure-1.1 shows a 5800 Series System. It includes a double column machine with accessories- grips and computers. Orders comprising this whole package i.e. machine with accessories is called a system order. Figure1.2 shows a similar grip. These grips (and other accessories) can also be sold separately from the whole system and such orders are the Over the Counter (OTC) orders. Figure 1.3 shows different accessories that can be a part of the system.



**Figure 1.3** - An Instron Two Column Machine with Accessories

Currently, Instron holds an inventory value of \$4million in the EM area with inventory control based on Distribution By Value (classifying products into categories A, B, C and D according to their cost) and ITW's policy of having a maximum of 2 months-on-hand demand. However, many aspects are neglected while determining their inventory policy- such as demand variability, percentage of lost sales, holding costs and customer expectations. Thus, there is a certain opportunity to scientifically determine the inventory levels taking all the significant factors into account and improving the customer satisfaction by fulfilling more orders as well as minimizing inventory levels.

## 1.2 Background

Generally, Instron's finished products can be classified into two categories: systems and OTC. In the electromechanical division, both of these categories exist and share a common inventory. Due to the demand variability, the management has decided not to base the inventory control on predicted demand but to switch to a pull production strategy in order to allow production to reorder parts only when finished goods are "pulled" away from the system. The physical implementation of pull production is achieved through the use of Kanban<sup>2</sup> cards for some of the purchased parts and components, and by not stocking some inventory items at all. However, Kanban is not available at the level of final finished goods level yet and has been implemented only 40% at the part level. Some finished goods are currently being replenished according to minimum level reports being generated through the internal inventory management system. This means that the goods are replenished only when a report is run and hence they are more prone to inaccuracies. Some other goods are being replenished by visually seeing on the floor if the quantity drops below a minimum mark, triggering a development order by the area manager. This method too can be inaccurate.

No records for lost orders are kept. Thus, it becomes difficult to determine which item causes the order to be lost. The available data shows only the orders which were fulfilled and hence, it acts as a barrier in determining the optimum inventory level since the actual demand will be underestimated.

The suppliers can also overlap i.e. one item can be bought from two different suppliers. This complicates the case further since there will be two lead times for the same product.

Finally, the final lead time to customer is also hard to determine due to a system audit which takes place on certain products and takes about a day to complete.



**Figure 1.4** – The finished goods stocking area

Currently, sometimes during peak demand, the factory floor gets clogged with the unfinished machines. Also, occasionally, when a whole order is made to wait longer, it gets cancelled, even if just one item was not available.

The markets of OTC and system orders have their own special requirement. While on the system side, the customers are more relenting and are willing to accept larger lead times, the OTC market is more demanding. The customers prefer expedited delivery since they are just waiting for one component in their system. Hence, the OTC market is very competitive.



The system market is usually more relaxed as Instron machines are expensive and they come as a capital purchase for their customers. Hence, the customers understand the large lead times for the machines. For a capital purchase, customers themselves need time to get the money sanctioned from their own organization. This too helps to mitigate the dissatisfaction due to high lead times. For the same reason, Instron starts building the product as soon as it gets an order. However, it does not ship it until all the bills have been cleared.

The systems market helps the OTC market by ensuring that customers buy only Instron accessories which are specifically designed for their machines. Easily available cheaper duplicates require extra adaptors and are not backed up with warranty. Despite this, customers want lowest possible lead times in the OTC products.

## **1.3 Significance of the Problem**

The significance of the project for Instron and the contribution of this study to the literature in the field of inventory management are shown in the following paragraphs.

### ***1.3.1 Significance of the Project***

The number of items and parts concerning the Assembly Department is around 1000. In this situation, a great waste of time and money can easily be caused by overstocking. On the other hand Instron's responsiveness to customer demand is identified as an important goal in order to maintain competitive advantage. The optimization of the control parameters is thus critical at the accessories area at Norwood. In order for the strategy to remain optimal in the future, the control parameters must be adjustable accordingly to variations in the product line and in the demand. It is also important that the proposed inventory strategy is easy to apply for the planners and the workers of the facility to properly control the stocking of so many items. The impact of proposing an effective inventory control strategy consists in improved production efficiency and better competitiveness on waiting times which is especially important for the OTC market.

### **1.3.2 Significance of the Study**

This work considers the case of low-volume high-mix inventory systems where customer orders may require several different products (i.e., high customization between products and hence demand between different products is correlated) and the shipment of those items cannot be split. The time delay seen by the customer is the performance measure of concern and the customer impatience is modeled and taken into account: whenever one or more items belonging to an order are backlogged, the customer is quoted a waiting time which is as long as the slowest item's lead time. As the waiting time increases, a customer is less prone to make the order. A continuous review model is proposed using historical sales data rather than using forecasted demand.

Interdependent demands frequently arise in real life multi-item inventory systems. The dependencies of demands for different inventory items may be implied by product options or kits. When the manufacturing lead times for some accessories are long or when customer order assembly time is small, the configuration of a proper mix of items is critical to ensure their availability with the desired probability and avoid order fulfillment delays. Ignoring correlation in the demand when present may lead to two possible consequences: stocking more than necessary or not being able to provide the desired service level. It is demonstrated by R. Zangh that this assumption leads to an overestimate of the total time delay when items are actually correlated [2].

Unfortunately most inventory models on time delay in the literature assume one-item orders. The resources available in the literature which consider interdependence in the inventory planning can be split in two main categories:

- Studies about joint replenishment take advantage of the correlation of the demand to minimize the ordering or setup costs and transportation costs. Unfortunately these techniques are not useful when items are provided by many suppliers. As described in the Introduction, for what concerns the case studied here, accessories are both manufactured in-house as well as ordered from a large number of outsider suppliers.
- A small number of studies describe similar problems but under different conditions. In particular some of them assume that parts belonging to the same

order can be shipped separately to the customer if some item is not immediately available. Other works consider other inventory control models.

### ***1.4 Review of Prior Instron Projects***

In the past ten years three MIT graduate students have completed research internships at Instron working on inventory control and operations management. The theses of D. Wheeler, G. Caterino and H. T. Nguyen are outlined below.

The purpose of Wheeler was to optimize the EM grip inventory by applying queuing theory, optimization techniques, supply chain rationalization and simulation models [3]. In particular the author, together with a project improvement team, achieved a thirty-percent reduction of the inventory for the 56 EM grips belonging to the Instron product line at that time. They implemented a pull production in the grip assembly job shop by setting up stock shelves for finished goods and components within the shop from which the parts were removed to fill the orders. When the level of finished goods drops below a specified quantity (the reorder quantity) the mechanic is signaled to replenish it. Moreover as the components to build the grips, which are drawn from the bins on the shelves, drop below the reorder point, the planner receives a signal and replenishment orders are placed. Reorder quantities and lot sizes for the finished grips and some components were provided by the Economic Order Quantity (EOQ) and the continuous review (Q,r) models. These models were applied on the most significant components which had been identified by applying the Distribution By Value (DBV) technique [4]. Items were classified as belonging to three different Classes (A, B and C). The most valuable components (Class A and B) were placed under the Q,r control policy, while reorder quantities and reorder points for items belonging to Class C were set respectively to one year's supply and six months' supply for each item.

The second thesis objective was to improve the responsiveness and flexibility of the assembly process applying elements of Lean Manufacturing [5]. With the use of Kanban

control in assembly, daily production schedules based on demand rate and decision rules to guide the work process, the assembly throughput times have been reduced by 40% on average in the final assembly operations. Changes to the physical assembly environment have been made in order to increase flexibility of the output. The author proposes an inventory policy to coordinate in-house inventory levels with manufacturing demand and improve the coordination with external suppliers. The policy, similarly to Wheeler's work, is based on a  $(Q,r)$  model and DBV and is tailored on a small number of finished good items (three selected product families). Its application on a pilot process showed a 15% reduction in the required floor space for an equivalent manufacturing output.

Nyugen in his work has tried to improve the service level by implementing lean initiatives in the plant [6]. Root cause and Value chain analysis were carried out in the plant to find opportunities for improvement. A material replenishment model was proposed that would help the company effectively pull parts from the suppliers. Lot sizes were determined according to extended economic order model quantities adjusted using Lagrange multiplier to account for multiple parts being manufactured at the same time. For the inventory control, continuous review policy is proposed for the EM business so that low safety stock can be kept and probability of stock out can be reduced.

In the next sections, the problem has been cleared defined qualitatively and quantitatively. Literature review for the work has been summarized in the next section. It highlights all the text that was helpful in understanding and interpreting the problem better. Next, the methodology to study the problem has been introduced which introduces the thought process used to develop the approach and then the steps that were followed, how data was collected and how it was interpreted. Finally, the problem was solved using the method highlighted in the above mentioned section and results obtained. These results after proper validation are discussed in the results section with some recommendations.

## 2. Problem statement

The project goal, shared among the four group members' theses, is the definition and implementation of an inventory control framework for the EM accessories stored in the Norwood facility. The result of this work is enabling the inventory planners of the Configuration Department to stock the optimal mix of accessories in order to guarantee a satisfactory service level to the customers and minimize the inventory cost.

### 2.1 Project Objectives

The project specifications provided by Instron are listed below.

- 1) Analyze the accessory level offerings based on customer demand and sales volume.
- 2) Determine finished goods inventory level for each accessory.
- 3) Develop and implement an internal finished goods replenishment model based on a *pull* strategy.
- 4) Coordinate with Supply Chain group to insure Kanban quantities support for the finished goods model.
- 5) Identify and procure any needed tooling.
- 6) Determine and implement any layout changes.
- 7) Measure and monitor results.
- 8) Make it visual and involve factory employees.
- 9) Identify key performance indicators.

## ***2.2 Designing the Optimal Inventory Policy***

In order to meet the specifications, the problem has been modeled and its critical elements have been identified.

A first challenge for this project comes from the large amount of accessories to control: more than 800 finished goods concern the Configuration Department and include grips, fixtures, faces, extensometers, couplings, adaptors, computers etc.

Some of them are assembled in the Norwood facility, while some of them are purchased parts or assemblies. The large number of components that constitute each finished item and the large number of vendors that supply Instron represent a further source of complexity for the analysis.

In the previous theses performed at Instron, a simplification of the large amount of parts considered was provided by Distribution By Value (DVB) and 80/20 techniques, which are described in Chapter 4, allowing the authors to focus on the most significant ones in terms of value or profitability. Since the 80/20 analysis is a currently widely used and appreciated tool within the company, the team decided to adopt it to perform an analysis of the demand, measuring volumes and profits.

As described in the Introduction, demand has two components: Systems and OTC. This allows the problem to be split in two separate analyses.

For OTC accessories customers expect immediate shipment. Since the OTC market is more sensitive to competitiveness, an effective control strategy is critical to provide customers with a satisfactory service.

The Systems market, instead, is characterized by longer waiting times expected by the customers and less external competitiveness. However all the parts of the machine must be shipped together, with rare exceptions, and if a part is missing the order is delayed. In fact most of the times customers cannot perform their tests if a part is missing, and in every case splitting the shipment of an order is costly and not desired by the company. In 2008 no more than 4% of the Systems orders got split and this percentage is meant to decrease.

While the OTC market can be analyzed considering individual profits and volumes for every item, an accurate model of the Systems demand should take into consideration the inter-correlation among products. This suggests that the demand analysis for systems should also account for the importance of an accessory as purchased together with critical items. The *Virtual Profit* is an index based on combined profits developed by the team to model the inter-dependence of the demands and it is presented in the paragraph 4.3.2.

Since the waiting time expectations for the two markets are different, the inventory levels for the same items must satisfy the two demands. The problem can be thus decomposed in two analyses for the different markets. Once both the stocking quantities are set for both demands a risk pooling strategy can be implemented by aggregating those results.

For both the markets, once the 80/20 analysis has provided a measurement of the criticality of the items within the product list, the proper inventory control policy for the items must be identified. Constraints to this project are given by the fact that the Norwood stocking capacity is limited and the inventory allowed by the Instron management is less than 2 MOH<sup>1</sup> (Months on Hand) for every item. Thus in order to maximize the customer satisfaction and so the profit, the basic strategy is implementing two different control policies for two different classes of accessories:

- The most critical items will be assembled or purchased to stock so that high service levels will be achieved.
- The less profitable items will be assembled or purchased to order, minimizing their inventory costs.

However the optimal division between items deserving to be stocked and items that will be made to order needs to be found. Another parameter to be set is the desired *Type I service level*, or percentage of customers that will be immediately served, for the first class items. Wheeler [3] suggests to favor the “80” items (those items that concur to the 80% of the total profit/volume or Correlation) and provide them with 0.95 Type I service level. Unfortunately there are two reasons why this is only a suboptimal solution:

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<sup>1</sup> Months on Hand = 12 (Average Inventory Value on Hand / COGS)

- The 80/20 curves usually show one or more steps in the distribution of volumes or profits, so that the division between most important and less important items is quite clear. This is also valid if the quantity measured is the Correlation. However the step does not necessarily occur at the 80% of the cumulative profit: its position can vary depending on the situation. Setting the threshold at 80% would lead only to a suboptimum.
- The 0.95 Type I service level was set accordingly to the Instron management which found it reasonable. However assigning a constant service level for all the make to stock parts is certainly not the optimal strategy.

This issue can be addressed designing an optimization problem which would allow splitting the items in the two classes in an optimal manner, setting at the same time the service levels for the for the first class items.

There are several factors that the problem must take into account. Firstly storing parts has a cost in terms of space, handling and cash blocking, in general referred to as *holding cost*, which has to be minimized. Moreover there are items which are more worthy to be stored than others because give a larger profit (on their own or being sold with other items). In order to consider the described issues the stock level for each item  $i$  will be determined by maximizing the expected total profit generated by that item. A model of the expected total profit is given by the expected revenue minus the expected total costs.

The expected revenue for each product can be found by multiplying its unit cost by its expected sales  $E(S_i)$ , which are a function of the demand rate and the number of items in stock. Note that the past and future expressions of the demand are not available since the sales lost because of the waiting time quoted are not registered and forecast is not used at Instron. Historical sales are the only information that can be found. For the purpose of this project we assume that the expected demand is equal to the past sales. The effects of this assumption are mitigated by the pull strategy that  $(Q,r)$  represents causing the actual demand to drive the inventory control once the control parameters are chosen.



Moreover, since customers are willing to wait a variable amount of time if the parts are not immediately available, sales are also function of the *delay acceptability*  $w_i$ , or the percentage of customers that would still buy the item if it is not in stock.

Currently, the production lot sizes or reorder quantities are determined based on their value and historical demand without taking into consideration the lead times. Though suppliers have a negotiated contract with the company, they are usually supportive of the lot size requirements. In order to guarantee the selected service levels to the customers, one of the components of the solution consists in making sure that these quantities are enough to satisfy the demand over lead time with satisfactory probability.

Finally, the raw materials control is evaluated. Based on the finished goods production, the raw materials inventory control has to be synchronized and the parts have to be available with high probability. An optimized policy is proposed in order to guarantee the necessary support to the finished goods replenishment model. The optimized policy requires knowing the suppliers' replenishment lead times; this requires data collection and accuracy. The raw materials control is evaluated by comparison with the current policy.

The resulting optimal strategy is evaluated in its costs and benefits: a simulation tool is designed in order to test and validate the control policy and compare it with the current situation.

In order for the finished good inventory policy to be implemented and utilized by the Instron workers in the future, the control parameters must be periodically computed and adjusted. For this reason the analytical tools used for this work are designed for reusability and robustness, as well as easiness of use and compatibility with the data and tools available at Instron. The tools must take into consideration adjustments for new products introduced in the product line and for dismissed ones. In fact the introduction of a new series of accessories with a partial substitution of some old one has occurred this year and can occur again in the future. The implementation of the strategy in the Configuration Department, including the physical arrangement of the stock bins and the Kanban cards, and the training of the workforce are part of this work, are part of this work, in order to guarantee that the strategy is correctly understood and continued.

## 3. Literature Survey

### 3.1 Introduction

Since our first contact with the problem, it was clear to us that its set of features and objectives made it a very particular challenge. The theory we learnt from classes and from Simchi-Levi et al. 2000 [7] guided us to the choice of a  $(Q,r)$  policy but the standard set of assumptions used to determine the parameters  $Q$  and  $R$  did not fit our problem. In particular the correlation between the demand of the various products, the fact that many items could be sold both alone and in a system order, the fact that a system order cannot be shipped unless all the items are available and the fact that customers have different expectations on acceptable lead times for different items required a new approach to solve the problem. Many of these challenges are somehow considered in literature but often with a different objective and anyway, to our best knowledge, they have been never considered together. In 3.2 we briefly discuss the vast literature about the  $(Q,r)$  policy which constitutes the basis of our work; in 3.3 we present papers which faces the demand correlation issue; in 3.4 we argue about the usage of some papers regarding the customers' expectation issue; in 3.5 some references about simulation are presented.

### 3.2 The $(Q,r)$ Policy

In those cases in which the inventory is reviewed continuously (in opposition to periodically) a heuristic control policy which has been well-studied in the last several decades is the so called “ $Q,r$ ” (sometimes also named  $r,Q$  or in other ways). The basic idea is that whenever the number of items held in inventory drops to or below  $r$  an amount of  $Q$  units of goods is issued to replenish the system. Hadley and Whitin 1963 [8] present an exact solution to the problem when there is a known penalty cost assessed

on each unit backordered and they provide, under some assumptions, two approximate iterative heuristic solutions.

During the following decades the Q,r policy has been extensively explored in literature, many of the original assumptions have been relaxed and many of its properties proved.

In particular important convexity results are given in Zipkin 1986 [9] and Federgruen and Zheng 1992 [10] and the existence of such results justify the research of optimum values.

Also, interesting convexity results are proved in Wang and Li 2007 [11] for the discrete demand and inventory case.

### **3.3 Correlated demand and the inventory management problem**

#### ***3.3.1 Correlated demand and job-fill rate***

Demand correlation among different items and its effect on inventory policies is a key aspect of this work. Even though it is common in real-life multi-item inventory systems, this phenomenon has not received a large attention in the existing inventory literature. We were able to find some papers related to the problem we are facing but none of them could directly be used in this case either because they pose different objectives or they are firmly based on a set of assumptions which does not apply to Instron case.

One of the first papers to focus on similar problems is Smith and Chambers 1980 [12]. In such work in fact it is introduced for the first time the concept of “job-fill” (in opposition to “part-fill”) rate criterion in this context. The paper deals with the determination of the appropriate collection of parts to be carried out to repair a machine. As in our case if only one part is missing the order cannot be completed (the machine cannot be repaired). In that case the cost associated with not being able to complete a given job due to unavailable parts is related to a longer downtime for the machine (the repairer has to go back to the warehouse and return on site again), in our case it is tied to the customer

unsatisfaction and the resulting risk of losing the order. Such problem was already known at the time as the “fly away kit problem” or the “submarine provisioning problem”, however these previous papers traded off shortages against part-fill rate instead of order-fill rate. Smith and Chambers [12] is then an interesting article but doesn’t consider all the issues present in our case because the correlation is not considered as the failures of different part types is assumed to be independent. However, other than for the “job-fill” rate criterion, [12] is very useful to us also for a theorem about the importance of ranking the items before considering an optimization problem.

Using Smith and Chambers’ “Job-fill” rate criterion, Zhang 1999 [13] studies the expected time delay in multi-items inventory systems. In such paper the demand is assumed to be correlated across items and customer satisfaction is measured by the time delays seen by the customers. As a result, an exact expression for the expected total time delay is derived. Also, it is shown that when items are actually correlated, assuming items are independent leads to an overestimate of the total time delay. This however assumes that the parts can be sold separately if some of them are not in stock. In this sense it is shown that demand correlation is in fact an opportunity that should be exploited. In our case, because an order cannot be shipped unless all the parts are available, the demand correlation is an issue.

### ***3.3.2 Correlated demand and joint replenishment***

The point of view presented in [13] is common to many other papers that deal with correlated demand. In fact many papers who consider demand correlation are focused on joint replenishments policies such as Liu et Yuan 2000 [14], Feng et al. 2007 [15] and Tsai et al. 2009 [16]. In particular [14] specifically considers the can-order policy for a two-item inventory system with correlated demands. Unfortunately joint replenishment doesn’t specifically help with the problems that Instron want to solve in its EM department and, even though it can still be beneficial, its usage would add a large amount of complexity and would allow very small benefits, if any. In fact, as regards items manufactured outside the company Instron has a very large number of suppliers and buys from each of them a very small amount of different products. Moreover, as regards items

manufactured inside the company, very small setup costs are involved and the assembly is mostly make-to-order. In other words in the papers which focus on joint replenishment the objective is reaching a balance between ordering costs, storage costs and stockout costs while in our case ordering costs are not significant. The same considerations about joint replenishments also apply to [15] and [16]. Specifically, [15] formulates the problem as a Markov Decision Process and focuses on joint replenishment and correlated demand, proposing a moving boundary based policy and comparing it to other control policies. Tsai et al. [16] instead propose a clustering algorithm to deal with demand correlation which is similar to a first possible solution, later abandoned, that we considered to solve our problem. Such paper claims that it is difficult to define the demand correlation between items, especially when the number of items increases and for this reason a clustering algorithm is proposed. Such algorithm is used to find an optimal clustering result which is used to determine the parameters of a can-order policy in presence of joint replenishment. The result is then tested through simulation and sensitivity analysis, two steps that are fundamental also in our approach.

### ***3.3.3 Previous work with different assumptions***

As said the literature which deals with correlated demand is relatively small and a good part of it is focused on joint replenishment which is not useful in our case. However, some papers are closely related in their intent to our work, although not directly applicable due to different assumptions. Hausman et al. 1998 [17] has very similar problem statement to our as it is said that the objective is to “configure a proper combination of component item inventories so that availability of component items is ensured at pre-specified levels to avoid order fulfillment delays”. Unfortunately this paper considers a periodic review order-up-to policy and so is not compatible with continuous replenishment. Anyway the paper contains some very interest ideas and some theorems and lemmas which can be considered also in our case. Very close to our objective is also Wang et Hu 2008 [18] which studies the application of a (Q,r) policy with budget constraints and optional components. The budget constraints, at least in the way they are formulated in [18], are not of primary concern in our case but the approach

proposed is still very interesting. Unfortunately, two of their assumptions are not verified in our case: it is not true that the payment is due at the time an order is placed (but this problem could be overcome) and most importantly it is not true that the customer will purchase a system without optional components when the optional components are out of stock. Optional components are in fact, in the majority of cases, necessary to use the Instron machine and no one would buy a machine without them.

### 3.4 Customer defection

In this work, the effect of customer impatience (or *defection*) on the inventory performance is studied. Two main contributions on this field are used as references: Gershwin et al. 2009 [19] and Veatch [20]. The main reason why this work investigates the customer impatience is that the number of orders filled (in literature *Type II Service level*) depends on how many customers would wait for a product if it were not in stock. In particular, the number of filled orders is the sum of the number of orders filled immediately plus the number of orders completed because the customers decided to wait and not to cancel the order once they were quoted a lead time.

In [19], a manufacturing firm that builds a product to stock in order to meet a random demand is studied. If a product is not in stock and orders cannot be met, customers are quoted a lead time that is proportional to the backlog, based on the production time. In order to represent the customers' response to waiting, a *defection function* - the fraction of customers who choose not to order as a function of the quoted lead time - is introduced. The defection function is then used to obtain the optimal production policy, which results in a hedging point form. One family of defection functions is studied, a sigmoid function of the form:

$$B(x) = \frac{1}{1 + e^{\gamma(x-\eta)}} \quad (3.1)$$

This expression for the defection function is then used to model the system behavior, and will also be used in this work. However, an additional important conclusion is that numerical results suggest that there is limited sensitivity to the exact shape of  $B(x)$ .

Furthermore, the precision of the defection function is limited by the intrinsic approximate nature of what it models, i.e. the customer impatience.

In [20] the same production model, in which the customer is quoted a lead time depending on production time and backlog, is presented as a “nuanced model” of customer behavior, compared to the two extreme models of complete backordering and lost sales, where all the customers either wait or not. One particular production model is considered: a continuous one-part-type, single machine model with Markov modulated demand and deterministic production is considered. Using this particular model, the impact of customer impatience is shown to be captured by one quantity, the mean sojourn time in the backlog states. As in [19], the optimal quantity has hedging point form.

Based on the particular model considered, Veatch shows that the effect of customer impatience can be captured by the only mean sojourn time in backlog, and this simplifies the problem of obtaining an optimal production policy. Given that the effect of customer impatience is captured by the above mentioned quantity, in fact, other simpler customer behavior models can be used, and still the optimal policy is reached.

This thesis analyzes a different model: only some of the products are produced in the factory floor, while most of them are ordered from suppliers. Moreover, the replenishment lead time is random and constraints on the reorder quantities have to be considered. Thus, the assumptions made in [19] and [20] are not valid any more, and the optimization problem is different. Moreover, the two papers do not present any attempt to shape the defection function in the actual industrial application. However, the analyzed work gave some useful insight into the modeling of customer impatience. The suggested sigmoid form is used in this work, and the limited sensitivity to the exact shape of the function is considered. Finally, this thesis considers the use of company-wide surveys in order to shape the defection function to the needed precision level.

### **3.5 Simulation**

Simulation has been used as a validation tool in this work. Monte Carlo is one of the simulation techniques used to validate our results. The principle behind Monte Carlo

simulation is that the behavior of a statistic in random samples can be accessed by the empirical process of drawing lots of random samples and observing the behavior [21]. However, care has been taken while generating customer demand. Truncated normal distribution is used to generate demand since it should not go negative in the cases when the coefficient of variation is high [22]. Coakley and Carpenter 1983 [23] have used Monte Carlo simulation to predict final system behavior when it cannot be directly predicted from the inventory models. They validate the model before running the simulation using constant values and matching them with theoretical results. Finally, they use the simulation results to analyze different conditions such as relaxing theoretical constraints and getting the inventory levels.

Jung et al. 2004 [24] have presented a method to determine safety stock levels, which further effect the customer satisfaction levels (service levels), using a computational framework for planning scheduling applications in realistic supply chains. They use simulations to optimize their results when faced with improving customer satisfaction, holding costs and production constraints. Inside the computation for optimization, repeated simulation of the supply chain takes place over the planning horizon, each with given Monte Carlo demand samples. Then, within each of these simulations, a series of planning and scheduling optimization problems are solved.

Grange [25] in his paper pays particular focus to demand distributions of slow moving items. He finds out the misidentifying demand distributions can have a detrimental effect on the fill rate leading to high and lower rates depending on over and under estimation of right tails. He also adds that multi-SKU inventory compensates misidentification by reallocating investment relative to the costs and expected demands of all the SKUs. We have thus, taken particular care in finding out the demand distribution in our case, as highlight in the methods section.

## **3.6 Conclusion**

The problem this work deals with is a particular one and a solution tailored for this case cannot be found in literature. Not many authors focused on demand correlation in multi-



items inventory systems and many of them consider a rather different set of assumptions thus being allowed to see it as an opportunity to be exploited using joint replenishment. A few papers which consider a similar problem statement are still not applicable to our case because they differ in some fundamental assumption such as periodic inventory review and optional nature of accessories. Also as regards the customer impatience issue the papers analyzed do not provide a univocal methodology to be used in our practical case but they contain very interesting ideas and results. Simulation was also found to be frequently used both as tool to find a solution and as tool to validate the result found with another method.

In conclusion our problem requires a new solution in order to deal with all its features but the existing literature constitutes a fundamental basis to our work with its ideas, theorems, reasoning and methods.

# 4. Methods

## 4.1 Choosing the right methods

The goals of this project are described in detail in chapters 1 and 2. One sentence summarizes them effectively: “having the right mix of products on the shelves at the right time”. As mentioned before, this involves searching an optimal inventory control and production policy by considering all the products together, especially taking into account the system orders, thus the correlation among items’ demands.

The significant number of items involved and the differences in their supply chains added high levels of complexity to the project. Not only do we want to have the correct “mix” on the shelf, but the implementation of the derived policies will differ depending on the product’s type and supply chain. Furthermore, using one’s own judgment on each SKU would not provide the company with a repeatable strategy. For these reasons, general and parametric methods always have to be used.

In addition to the optimal policies, important results of the project come from the analysis phase (demand analysis, correlation analysis, customer defection, 80/20). The produced documents, indeed, are important in providing the manufacturing, sales and marketing departments with sources of data which allow effective strategic planning. As an example, knowing which products are often sold together in the last two years, could suggest marketing already customized systems (composed of the products often sold together); if this operation is successful, the company could focus its investment in the inventory for a limited number of products, holding less risk associated with other products. Moreover the results of the analysis performed by the team and provided to the company find an application in the identification of products to discontinue because of their scarce profitability and importance within the product list.

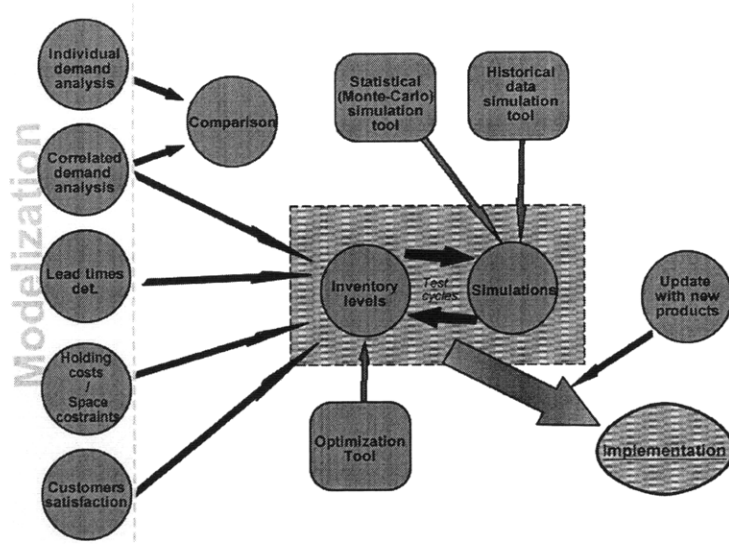
What is more, in each sub-issue addressed by this thesis, the purpose is not only identifying the optimum (optimal inventory control policy, optimal replenishment levels)

but also proposing the so called “*good enough*” solution. As widely happens in manufacturing and operations management, in fact, the application of systematically searched optimal policies holds a level of complexity that is not worth the investment. For instance, considering the optimal replenishment methods, agreeing with the suppliers on the optimal reorder quantities for a product could not be feasible or could involve additional investment, and using a QR policy implemented with Kanban cards, that are already used, would be more easily and quickly implementable than different policies that could guarantee a relatively small increase in expected profit.

In conclusion, the work described in this thesis is meant to produce data analysis reports and suitable solutions for the inventory control policies of a significant number of products. This chapter describes the steps that are undertaken in building the analysis reports, in designing the control policies and in collecting the necessary data for the policies to be implemented. The methods used in each step are briefly described in the following paragraphs and then explained in more details in the following chapters.

## **4.2 Main steps followed**

Figure 4-1 shows the main steps involved in the project. Every independent task is represented by a blue filled circle, while the developed software tools are represented by smoothed rectangles. The arrows indicate task scheduling requirements. As an example, let us consider the following tasks: comparison, individual demand analysis and correlated demand analysis. In order to perform the comparison task, the results from the individual and correlated demand analysis are necessary; thus, these two tasks need to be finished in order for the comparison task to be performed. The diagram is a modified version of the PERT diagram which does not show the duration of the tasks.



**Figure 4-1** Tasks Diagram

As previously mentioned, the main outcome of the project consists in data analysis reports and recommendations for inventory control policies. The most important reports are obtained in the steps *Individual demand analysis*, *Correlated demand analysis* and *Comparison*. In these three steps, demand analysis of all the involved products is performed, at first simply by volume and profit, and then considering how they correlate to each other. Finally the results are collected in a Comparison report, meant to underline the importance of the correlation. The step *Inventory level* involves designing the control policies, while the performance of these policies are estimated in the step *Simulations* and implemented in the step *Implementation*. The importance of these two final steps is highlighted by the orange box in the diagram.

The left side of the diagram shows the steps needed in modeling the system. In order to design the inventory control policies, the following information is needed: lead times for each product, profit and correlation analysis, holding costs, space constraints and a model of the customer satisfaction. All this information builds the model of the system, used to find the optimal solutions.

The remaining part of this chapter describes the goals of each task, the approach to it and the methods used.

## 4.3 Explanation of the tasks

### 4.3.1 Individual demand analysis or Pareto analysis

This task involves analyzing the orders placed in 2008 and 2009. The list of orders, together with the associated quantities and prices, is used to perform a demand analysis based on both profit and volume. The purpose of this analysis is to find the most important products and the least profitable ones. The results are useful to the company in showing the updated data on volume and profit made by the products during the last two years.

The *Pareto principle* (also called *80/20 principle*) is a heuristic principle that is often applied in analyzing profit and volume in operations management (the Pareto analysis). Applied to profit, it states that about 80% of the profit of a company is made by only 20% of the products it sells. The products belonging to that 80%, which are the most profitable ones, are called the *80s*, while the remaining products are the *20s*.

For the purpose of this analysis, the products are divided in six different categories: grips, fixtures, faces, coupling and adapters, compression anvils and anvil sets and other accessories. The first step of the analysis involves summing up the profits made by each product in all the orders and determining the total quantity shipped in each year. A report has been given to the supervisor, in which the most profitable items were identified through the Pareto analysis. In addition to this, the least profitable items were highlighted in the report: all those products which belong to the bottom 1% of the profit or were sold at most twice. This result is important to identify items eligible to be discontinued. However it does not provide a measurement of their criticality within the product list. The Correlation analysis, described in 4.3.1, provides a more accurate result.

For a more expanded discussion of the Pareto analysis, please see Chapter 5 in Diego Palano's master's thesis [26].

### **4.3.2 Correlated demand analysis and Comparison**

As mentioned earlier, the design of the optimal policy is complex because it has to encompass a very high number of different accessories that are often sold together in the system orders (when customers buy a machine and choose a set of accessories with it). Moreover, the above mentioned individual demand analysis is less accurate than necessary because it does not take into account the system orders.

As an example, two products X and Y can be considered. If X is an “80” item and Y is one of the lowest profit items, the individual demand analysis would suggest holding less inventory for item Y or even making it to order. By considering the system orders, however, we could find out that product Y is often sold together with X, and is less profitable because it is discounted or relatively less important. Holding lower inventory levels for item Y would then be a losing strategy, because it would block the orders of X and create additional profit loss.

In this project, the correlation between different products is considered in designing the control policies. The goal is obtaining a profit indicator which quantifies the profit made by each product if in stock, or quantifies the loss realized by not having it in stock for a given period of time. A MATLAB function, using the IBS reports with all the orders of 2007 and 2008, calculates how many times each product is sold with any other item and quantifies this expected profit.

New profit indicators were obtained considering the correlation, and a new analysis report was generated (step *Comparison*). This report shows what are the most profitable items and what are the ones which are still in the bottom 1% of the profit after considering the correlation. As mentioned in paragraph 4.3.1, this report completes the analysis of the items to be discontinued, together with the 80/20 report.

For a more expanded discussion of the correlation analysis, please see Serra master’s thesis [27].

### ***4.3.3 Lead time, holding costs and space constraints***

These three steps involved data collection, which is necessary to design the control policies. The data collection methods, including holding costs and space constraints, are further explained in chapter 4.4 of this thesis.

By working with the supply chain managers and using the IBS tracking system, at first we tried to obtain a list of lead time values for all the products involved in the project. The term “lead time” was used in a more general sense, indicating replenishment lead times for purchased parts, manufacturing run time for manufactured or assembled parts, and collecting time for catalog numbers that actually are a kit of items. In general, the term, lead time, indicated the total time needed for a product to be again on the shelf when required.

### ***4.3.4 Customer satisfaction***

In order to maximize the expected profit, the loss for a part not being on the shelf has to be quantified. Let consider the case, however, in which one particular SKU is not on the shelf. The customer would learn that a particular product was not on the shelf and that the total waiting time would be  $n$  weeks. Would he still go on with the order? And what if the order request was actually for a system including that product?

In general, there will always be a number of customers who will still buy a product even if the order cannot be fulfilled from stock and a longer waiting time is quoted. This percentage depends on the product and on the type of order, and is a function of the quoted waiting time. This function is referred to as “customer defection”. The literature background about customer defection is discussed in chapter 3.

Obtaining this quantity from the data or in any rigorous way is not feasible due to the following reasons:

- Lack of hard data about lost sales
- Customers have different interests, priorities, concerns
- Other reasons (human behavior, complex products interdependence)

Thus, a reasonable estimate is obtained through a survey directed to the sales people, who work on orders with the customers. The starting expression of the customer defection function is a sigmoid, as discussed from the literature, and the function is further shaped by asking general questions and looking for ranges of values through the survey. This function represents the percentage of customers still willing to wait depending on the waiting time that can be offered on one particular item.

For a more expanded discussion of the customer defection analysis, please see Chapter 6 in Diego Palano's master's thesis [26].

#### **4.3.5 Inventory levels**

This task involves designing the production and inventory control policies for both finished goods and raw materials.

Two main types of policies are used: make to stock and make to order. The less profitable items will hold lower service levels or be made to order, while for the remaining products stock levels are determined. The choice of the MTO or MTS policy for each item is based on optimizing the profit, and is described in 4.3.6.

The most suitable make to stock inventory control policy is the QR policy (or *reorder quantity*). One reason is that the inventory at Instron has always been managed through two quantities: the so called *minimum quantity*, corresponding to the safety stock, and the reorder quantity. Even if these quantities were obtained with rules of thumb, they are used to set a safety stock level and reordering when the levels go below the minimum quantities. Moreover, an increasing number of parts are being managed by *Kanban cards*, which is an automatic inventory replenishment method. When the inventory level reaches a minimum quantity, the corresponding card is put on a board and it will automatically trigger the order of a predetermined release quantity from the suppliers. This system is easily updatable once the new optimal values for Q (reorder quantity) and R (reorder point) are derived.

The reorder quantities are determined in such a way that they cover the demand over lead time with a probability of 99.87%, still satisfying eventual constraints on the lot sizes.



The optimal reorder points, on the other hand, are calculated from the lead times, the average demand, the values of  $Q$  and the desired service levels. While lead times and average demand are obtained in the data collection phase, the service levels represent our degrees of freedom in designing the policy. For the finished goods inventory, these levels were chosen by optimizing the profit, as described in 4.3.6. The raw materials inventory control, instead, is designed in such a way that the service levels are always high, in order to support the finished goods production.

For a more expanded discussion of the raw materials inventory control, please see Chapter 7 in Diego Palano's master's thesis [26].

#### ***4.3.6 Optimization***

The available degrees of freedom in designing the FG inventory control policy are given by the service level corresponding to each item (Type I service level, defined as the percentage of time the inventory for a certain item will not be empty, thus being able to meet demand) and whether each product will be made to stock or made to order (MTS or MTO).

These choices are determined by solving an optimization problem. The goal function is the total expected profit, defined as total expected profit coming from sales minus the inventory holding costs. The total expected profit coming from sales is calculated considering the correlation between products in the same orders (as described in 4.3.2), while the inventory annual holding costs per item are multiplied by the expected inventory levels in the QR policy.

The result of the optimization tool, implemented in Matlab, is a list of optimal service levels for all the items. If the optimal service level for a particular product is lower than a certain limit than the final suggestion for it will be a make to order policy.

For a more expanded discussion of the finished goods policies optimization, please see the next chapters.

### **4.3.7 Simulation**

An important step in studying the optimal control policies is the simulation phase. It allows us to test the designed strategy in order to check its feasibility and to estimate its performance measures (actual service level obtained, months on hand of average inventory).

The simulation tools are used both as design aid and as final performance measurement that helps in selling the proposed recommendations. The simulations are implemented in two different ways: at first simulating random demand with a discrete probability distribution with the actual mean and standard deviation (plus intra-quarter growing average), then by using the actual historical data. The former tests the policy for robustness with a more general background; the latter shows a comparison between the results of the proposed policy and the current one.

For a more expanded discussion of the simulation of the proposed policies, please see Samarth Chugh's master's thesis [28].

## **4.4 Data collection methods and IBS**

Most of the tasks undertaken in modeling the system involved hard data collection from the databases of the company. Referring to the diagram in picture 1, these tasks are:

- Individual demand analysis;
- Correlated demand analysis;
- Lead times determination;
- Holding costs / Space constraints;
- Customer satisfaction;
- Historical data simulation tool;
- Update with new products.

The holding costs are obtained from the operations manager and head of manufacturing and through some financial research on cost of capital; the space constraints are estimated talking to the managers and exploring the factory floor. The information about the new

products (new item numbers, discontinued items, updated demand forecast) was obtained from to the engineers in charge of the corresponding projects.

The model of customer satisfaction is firstly defined based upon literature and suggestions from the operations management. Then, the model is shaped and refined through a company-wise survey, filled by the sales department and the field engineers, who are the ones involved in the customer satisfaction aspect of sales.

All the remaining tasks involve collecting data from Instron's databases:

- previous years' sales
- product types
- inventory locations
- costs and prices
- replenishment lead times
- manufacturing run times and set-up times
- current reorder points and quantities

The necessary information is collected through IBS. IBS is an Instron database management system that tracks all the information associated with orders and products. For each order placed by customers, IBS contains order number, dates, quoted lead times, standard costs, gross price, discounts and a number of other entries. For each product, IBS contains item number, bill of materials, information about suppliers and planners, current inventory levels and limited inventory level history, lead time and a number of other entries.

IBS is used in all the departments in the company. The sales people, when dealing with customers, use IBS to get the expected lead times, to check what is available in stock, to check prices and costs and to handle orders. The employees working in the factory floor update it when parts arrive from suppliers, when products are shipped, when changes are made to the orders, when WIP inventory is used and a part is assembled and in several other cases. Moreover, all the other employees often use IBS to get required information for analysis purposes or to update it.

In order to collect the needed data, reports are automatically generated by IBS. IBS can be queried with a list of items or orders, and the required information is written on Excel spreadsheets. The result is that every analysis or manipulation which starts from the generated spreadsheets can be easily repeated and updated by using the same type of queries.

## **4.5 MATLAB implementation and reusability**

### ***4.5.1 The need for a tool***

The goal of the project at Instron is not only to provide a numerical solution to the problem of which control policy and which parameters should be used. Also, a fundamental goal is to provide a long term solution framework, so that, year after year and quarter after quarter, a new numerical solution can be computed and used. In fact one has to consider that every product has a certain life cycle and that the demand for each of them changes over time. Therefore, it is clear that the “determination of the right mix” is not something that can be determined once. On the contrary, a regular update of the safety stocks levels and inventory control policies parameters is necessary.

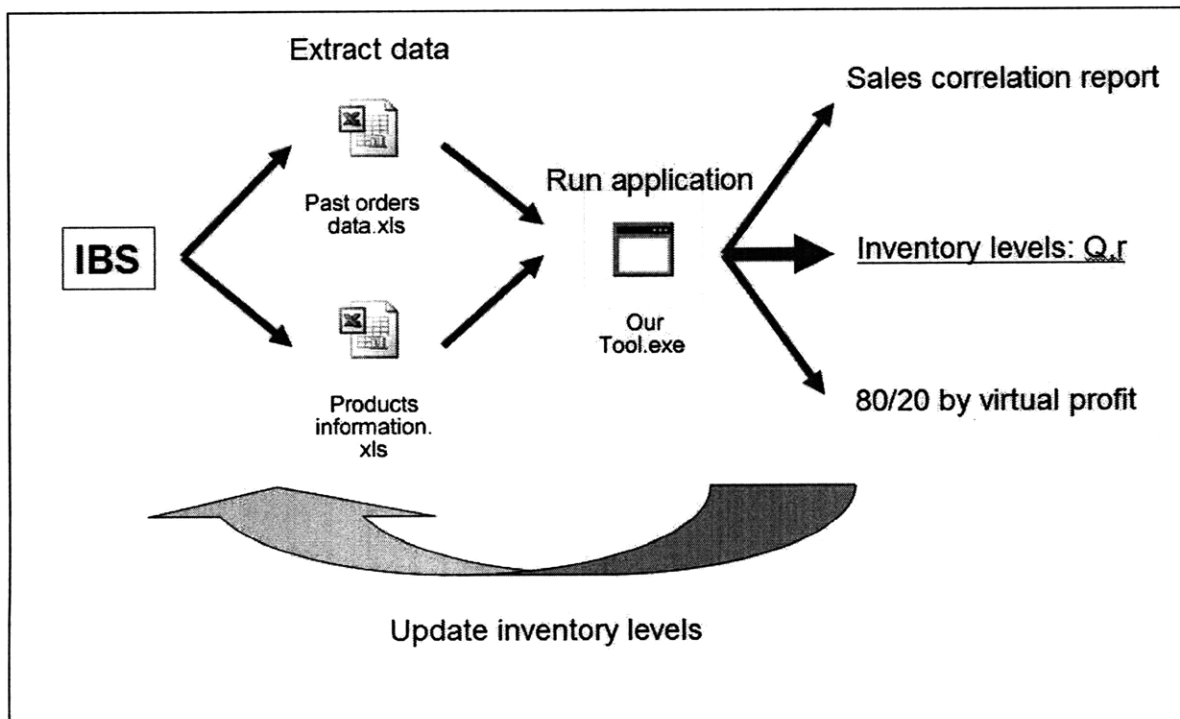
For this reason, since the beginning of the project the research team focused on creating a tool that could be used in the research and that then Instron could use in the future to make the calculations and update the policies regularly.

### ***4.5.2 Reusability***

The way we see the solution framework is depicted in Table 4.1. On a periodic basis (the choice of the frequency is discussed briefly in the next paragraph) Instron personnel will update the inventory levels. In order to do this, they will export all the relevant past sales data from IBS (the ERP software they are currently using) to an Excel file using a template that we built in IBS. Then, in a similar way, a list containing the lead times, the lot sizes and other information regarding the items will be extracted from IBS. Finally

these XLS files will be put into the same folder as our software tool (an EXE file) and by just running it a solution will be computed.

The output will be composed of three files. The first one is an Excel file containing the information that should be used for the Kanban cards, that is to say the reorder quantities and the reorder point that has just been determined. The second file is a *Correlation report* that is to say a description of the items that were most often sold together which is useful for Instron personnel to understand the demand and what drove the suggested inventory levels. Finally, the third output is an *80/20 report* in which the items are divided by category and ranked by their virtual profit. Also this report will help to explain to the people the re-order quantities determined by the tool and it will also suggest which items can be suppressed without losing, both directly and indirectly, much profit.



**Table 4-1** Reusability scheme

### ***4.5.3 Frequency of stock determination***

There is a trade-off in the frequency with which the inventory levels should be re-computed. In fact, on one hand the higher the frequency with which the inventory is re-determined, the better the inventory levels will theoretically perform because they will use the most recent demand information. On the other hand, re-determining the levels involves a certain effort from Instron staff and represents a cost that can balance the advantage of using more recent data. To determine the new levels in fact some data has to be gathered as described above and the computation has to be started. Then the resulting suggested reorder quantities has to be compared with the ones currently in use. If an “R” needs to be updated, then the Kanban card currently used for that item must be reprinted and substituted on the bin.

As seen, a trade-off exists and the correct time does determine new levels depend on the effort necessary to physically update the inventory levels. As a first guess, we think a frequency of 3 or 6 months seems reasonable, unless some of the determining factors (the demand or the lead times for example) will at some point drastically change.

### ***4.5.4 Matlab implementation, reusability and flexibility***

The tool described above is built in the Matlab environment and then compiled as an executable file. The choice was suggested by our familiarity with such environment and its power and abundance of mathematical functions. As regards the part of the code which deals with data crunching a C code would have probably been faster but in such a language the optimization part would have been harder to code and, overall, the time required to build the tool and test it would have been much longer. Because in our case the quickness with which the tool was to be built is very important while the computation time required for every run is not particularly significant (as seen the tool is going to be run a few times per year), the choice of Matlab seems to be the best one.

Moreover, Instron owns many Matlab licenses for other reasons so such software is and will be available to the company without any added cost. This is an important issue because, even though we want to give an “easy to use” – “black box” solution, we also

want to provide the source code that could be checked and modified in the future and while to run the exe file Matlab is not necessary, to modify the source code is.

# 5 The control policy

## 5.1 Introduction

The proposed control policy is based on the reorder point policy  $(Q,r)$ . In chapter 5.2 the  $(Q,r)$  policy and its assumptions are presented and the type I and type II service levels are defined. In 5.3 it is discussed the way we determined the reorder quantity  $Q$  in this case. In 5.4 we introduce the optimization problem that will determine the reorder level  $R$ . The exact formulations of such problems and its solutions are presented in chapters 6 and 7.

Note that the main reference used for the  $(Q,r)$  model is Hadley and Whitin book (1963) 29.

## 5.2 The $(Q,R)$ Policy

### ***5.2.1 Definitions and Type I Service Level***

The proposed approach is based on the  $(Q,R)$  model, a continuous review inventory model which requires the following basic assumptions as stated by Wang and Hu (2007) 30; a discussion of these assumptions in relation to our case can be found in 5.2.4).

1. Demand comes from a stationary process with known mean  $\mu$  and standard deviation  $\sigma$ .
2. Demand is independent in non-overlapping time increments
3. The demand over lead time does not exceed the reorder quantity  $Q$
4. Lead time is constant (however extensions of the model exist where this assumption is relaxed)

An internal order of constant size,  $Q$  is placed whenever the inventory level drops to or below the reorder quantity  $R$ . The quantity  $Q$  can be determined using, for example, an



EOQ (Economic Order Quantity) model which would consider the tradeoff between set-up or shipping cost and inventory holding cost. Note that in some cases the aforementioned tradeoff might not be evaluated because either the holding costs or the set-up/shipping costs are difficult to determine. Moreover constraints from the supplier about the order quantities might exist and limit the choice of  $Q$ . In any case,  $Q$  should be large enough so that the hypothesis 3 of the  $(Q,R)$  model is satisfied. Given  $Q$ ,  $R$  is determined in order to cover the lead-time demand with a certain probability. It is now convenient to introduce the concept of Type I service level which we will simply refer to as service level from now on:

The service level  $\alpha$  is defined as the probability of not stocking out when there is an order event.

For a  $(Q,R)$  model, the probability of stocking out during the lead time is given by the probability that the demand  $x$  during the lead time  $L$  is larger than the reorder level  $R$ .

Therefore, defining  $f_L(u)$  as the probability density function of the demand over the lead time, the probability of stocking out can be written as

$$1 - \alpha = \int_R^{\infty} f_L(x) dx \quad (5.1)$$

If the mean of the demand is  $\mu$  and its standard deviation is  $\sigma$ , the demand over the lead time has mean  $\mu L$  and standard deviation  $\sigma\sqrt{L}$ . Now, if we assume that the demand is characterized by a certain probability density function, we might solve the integral (5.1). In particular we assume the demand to be normally distributed; therefore to the four assumptions of the  $(Q,R)$  model listed before we add a fifth one:

##### 5. The demand is normally distributed

Then the probability of stock out can be expressed as:

$$\int_{x=R}^{\infty} \varphi_{\mu L, \sigma \sqrt{L}}(x) dx = 1 - \int_{-\infty}^R \varphi_{\mu L, \sigma \sqrt{L}}(x) dx \quad (5.2)$$

where  $\varphi_{\mu, \sigma}$  represents a normal probability density function with mean  $\mu$  and standard deviation  $\sigma$ .

Naming  $z$  the safety factor, let set:

$$R = \mu L + z \sigma \sqrt{L} \quad (5.3)$$

And substituting the expression for  $R$  in (5.3) into equation (5.2) we get

$$1 - \int_{-\infty}^{\mu L + z \sigma \sqrt{L}} \varphi_{\mu L, \sigma \sqrt{L}}(x) dx \quad (5.4)$$

Applying the definition of cumulative distribution function (abbreviated as cdf), and calling  $\Phi_{\mu, \sigma}$  a normal cdf with mean  $\mu$  and variance  $\sigma$ , (5.4) can be rewritten as

$$1 - \Phi_{\mu L, \sigma \sqrt{L}}(\mu L + z \sigma \sqrt{L}) \quad (5.8)$$

Finally, using  $\Phi$  to indicate a standard normal cdf, (5.8) becomes

$$1 - \Phi(z) \quad (5.9)$$

In conclusion, using (5.1) and (5.9) the **Type I** service level  $\alpha$  using the (Q,R) policy can be expressed as

$$\alpha = \Phi(z) \quad (5.10)$$

### 5.2.2 Expected inventory level

The expected inventory level is one of the most important characteristics of any inventory system. In particular, we want to use it in our optimization to evaluate the holding cost of a given set of  $Q$  and  $R$ . Unfortunately, the exact expected inventory level is not easy to

compute, hence several approximating procedures have been proposed in the literature. A discussion about some of these approximations and an attempt to identify the “best” one can be found in 31. Such paper reports that the exact expression under deterministic lead times, non-negative  $R$  but otherwise fairly general conditions is:

$$\frac{1}{Q} \int_R^{Q+R} \left\{ \int_0^y (y-x) f_L(x) dx \right\} dy \quad (5.11)$$

In 31 equation (5.11) is also modified to be applicable for both positive and negative values of  $R$  as

$$\frac{1}{Q} \int_{\max(0,R)}^{Q+R} \left\{ \int_0^y (y-x) f_L(x) dx \right\} dy \quad (5.12)$$

31 proposes its own approximating formula for (5.12) but also concludes that the most common approximation to the average inventory level given in 29

$$\frac{Q}{2} + R - \mu L \quad (5.13)$$

is more robust than the literature appears to imply. In this work we use (5.13) to evaluate the expected inventory level. As part of future work it might be interesting to evaluate which performances might be achieved using other approximating formulas in the framework presented in this work.

Finally, note that an equivalent expression of (5.13) can be obtained by substituting the expression of  $R$  (5.3) into (5.13):

$$\frac{Q}{2} + z\sigma\sqrt{L} \quad (5.14)$$

### 5.2.3 Type II Service Level

Let us now introduce the concept of **Type II service level or fill rate**, defined as the **percentage of demand that is immediately met from inventory**.

For a (Q,R) model, the following approximation for the Type II service level can be made. Defining  $\beta$  as 1 minus the fraction of demand not met from inventory we can express  $\beta$  as:

$$\beta = 1 - \frac{\int_{x=R}^{\infty} (x-R) f_L(x) dx}{Q} \quad (5.15)$$

The integral in (5.15) is known as *partial loss function*.

Suppose demand over the lead time has a normal distribution with mean  $\mu L$  and standard deviation  $\sigma\sqrt{L}$ , setting  $R$  as (5.3) and following a reasoning similar to the one used for determining the Type I service level, the partial loss function can be expressed as:

$$\int_{x=R}^{\infty} (x-R) f_L(x) dx = \int_{\mu L + z\sigma\sqrt{L}}^{\infty} (x - \mu L - z\sigma\sqrt{L}) \phi_{\mu L, \sigma\sqrt{L}}(x) dx \quad (5.16)$$

And considering the standard normal cdf and pdf (5.16) can be written as

$$\int_{x=R}^{\infty} (x-R) f_L(x) dx = \sigma\sqrt{L} [\phi(z) - z(1 - \Phi(z))] \quad (5.17)$$

Finally, using expression (5.17) in (5.15) we obtain

$$\beta = 1 - \frac{\sigma\sqrt{L} [\phi(z) - z(1 - \Phi(z))]}{Q} \quad (5.18)$$

### **5.2.4 Discussion of the assumptions**

With reference to the assumptions presented in 5.2.1, in reality we can expect that some of them will not be verified.

For example it is very likely that the assumption of stationary demand is not respected and also that the future demand is not known. In order to deal with this fact our approach is to assume the future demand to be equal to the one which comes from historical sales modified by the user if he can estimate some changes will happen. This means that in its practical implementation our policy will take into consideration any information about a demand mean shift for a single product or for the aggregate sales that the user will be confident to use. A further discussion of the way this is implemented can be found in chapter 9.

Another assumption is that the demand over lead time does not exceed the reorder quantity  $Q$ . In this case the chosen approach is to take into consideration a  $Q$  large enough to make this assumption verified with a very high probability (as discussed in 5.3). Incidentally, a large value for  $Q$  is practically appreciated by the company as it reduces the number of jobs thus making the production scheduling simpler (complexity of operation is not explicitly considered in the proposed mathematical model).

Finally, for our project no historical data was available about actual lead times. Also, because many products are internally manufactured, the lead time actually depends on the production capacity. Considering that a model of the manufacturing capacity is not considered here and that a stochastic distribution of lead times cannot be researched, a best estimate of them is used in practice.

### 5.3 Determining Q

The reorder quantity Q is often determined considering the trade off between holding cost and the set up (for manufactured parts) or ordering (for purchased parts) cost. A basic approach in this sense is represented by the EOQ model but many other models are discussed in literature and used in practice. In our case however the set up costs were either not available or not significant and the shipping costs hard to determine. When information about set up cost was available a lot size Q' was already determined and stored in IBS and such quantity is taken into consideration. In any case, Q is chosen large enough to satisfy with a high probability the demand over the lead time. So,  $Q_k$  is determined as

$$Q_k = \max \left[ \text{ceil} \left( \mu_k L_k + 3\sigma_k \sqrt{L_k} \right), Q'_k \right] \quad (5.19)$$

## 5.4 Determining R: the optimization problem

The (Q,R) policy assumes that a service level is chosen by the user and the corresponding reorder level  $R$  is computed. However in some cases, it is of interest to consider different service levels for different products taking into consideration their different importance and the holding cost to store them. In these cases, an optimization problem can be formulated, and by solving it, the service levels and, as a consequence, the reorder levels can be found.

The goal function to maximize in order to optimally determine the stock level for a given item  $i$  is the expected total profit generated by that item.

The expected total profit can be calculated as the expected revenue minus the expected total costs.

The expected revenue for each product can be found by multiplying its unit price times its expected sales  $E(S_i)$ , which are a function of the demand rate and the number of items in stock. The considered item and the ones that are ordered in the same orders must in fact all be in stock and available. Note that the past and future expressions of the demand are not available, and only the past sales data are provided. In the following pages, we are assuming that the expected demand is equal to the past sales. Moreover, since some customers are willing to wait if the part is not in stock, sales are also function of the delay acceptability  $w_i$ , that is to say the percentage of customers that would still buy the item if it is not in stock and they have to wait the estimated waiting time.

Systems orders contain many items that cannot be shipped separately usually because they cannot be used without each other. Therefore another factor that influences the expected sales is the availability in stock and delay acceptability of the items that are ordered together with item  $i$ . The holding cost and the production cost represent all the costs considered for this problem. The expected holding cost is given by a unit holding cost  $h_i$  multiplied by the expected inventory  $E(I_i)$  as defined in 5.2.2.

The production cost depends on the number of items produced, which can be greater than the number of items sold. However assuming that items are not perishable while they sit

in stock and do not become obsolete (which are reasonable assumptions for the Instron products), when the number of produced items is greater than the number of sold items, they can be stored and sold in the next period (year or quarter). Therefore for those items there will not be a production cost in the following period.

In order to simplify the reasoning, the production cost is ascribed to the period in which the item is sold. This does not change the total production cost.

The unit profit from a sale  $p_i$  can be expressed as the difference of the unit price and the unit production cost. Since the standard values provided by Instron change year by year and may contain errors which are adjusted, introducing discounts or increment of price when the order is registered, the unit price and cost are determined considering their averages in the analyzed period and including discounts and other adjustments. This provides robustness to the method to errors and variations of price.

The expected total profit model for every item  $i$  can be written as

$$E(S_i)p_i - h_i E(I_i) . \quad (5.20)$$

Therefore the objective function becomes:

$$\max \sum_{i=1}^I E(S_i)p_i - h_i E(I_i) \quad (5.21)$$



# 6 Global Optimization Approach

## 6.1 First Formulation

### 6.1.1 The formulation

A first approach to formulate an optimization problem is described below. The formulation considers all the aspects of interest and makes one extra assumption (the probability of stock availability of every product is stochastically independent with the one of the others). Unfortunately it results in a single complex optimization problem which is hard to solve when a large quantity of products and orders must be considered. The approaches presented in 6.2 and 7 results instead in a solvable problem, although with some differences that will be analyzed later.

Let us define  $U$ ,  $T$ ,  $n_{i,j}$  as

$T$ : is the number of items

$U$ : is the number of orders

$n_{i,j}$ : is the number of times item  $i$  is sold in the order  $j$

The expected sales for item  $i$  can be written as

$$E(S_i) = \sum_{j=1}^U n_{i,j} P(\text{sell } j) \quad (6.1)$$

Where  $P(\text{sale}_j)$  is the probability that the order  $j$  is shipped. Considering that an order  $j$  can be shipped only if all the items  $m$  which are part of it ( $m \in \Omega_j$  where  $\Omega_j$  is the set of items requested in the order  $j$ ) are available and assuming that their availability is stochastically independent then (6.1) can be rewritten as:

$$E(S_i) = \sum_{j=1}^U n_{i,j} \prod_{m \in \Omega_j} P(m \text{ is available}) \quad (6.2)$$

However, because the probability that an item  $i$  is available when an order arrives is what we defined as  $\alpha_i$  then we can rewrite (6.2) as

$$E(S_i) = \sum_{j=1}^U n_{i,j} \prod_{m \in \Omega_j} \alpha_m \quad (6.3)$$

Considering equation (6.3) the objective function (5.21) can be rewritten as

$$\max_{\alpha} \left\{ \sum_{i=1}^T \left[ \sum_{j=1}^U (n_{i,j} \prod_{m \in \Omega_j} \alpha_m) p_i - h_i E(I_i) \right] \right\} \quad (6.4)$$

Equation (6.4) assumes that every item must be in stock in order for an order to be shipped. However there is a percentage of clients willing to accept the production lead time if a part is not in stock and this is what we modeled with the customer impatience curves (see [Palano] for more details). Therefore the probability that a customer buys item  $i$  is given by the probability that “item  $i$  is in stock” plus the probability that “the customer accepts the lead time given that the item is not in stock”. Therefore in place of  $\alpha_i$  we can consider the term

$$\alpha_i + (1 - \alpha_i) w_i \quad (6.5)$$

Using (6.5) equation (6.4) becomes

$$\max_{\alpha} \left\{ \sum_{i=1}^T \left[ \sum_{j=1}^U (n_{i,j} \prod_{m \in \Omega_j} (\alpha_m + (1 - \alpha_m) w_m)) p_i - h_i E(I_i) \right] \right\} \quad (6.6)$$

Substituting in (6.6) the expression (5.14) which gives the expected inventory level when QR policy is used, we get

$$\max_{\alpha} \left\{ \sum_{i=1}^T \left[ \sum_{j=1}^U (n_{i,j} \prod_{m \in \Omega_j} (\alpha_m + (1 - \alpha_m) w_m)) p_i - h_i \left( \frac{Q_i}{2} + z_i \sigma_i \sqrt{L_i} \right) \right] \right\} \quad (6.7)$$

Moreover, assuming the demands are normally distributed, as shown in (5.10) we have that  $\alpha_i = \Phi(z_i)$

Also, we can express the holding cost as the product of the cost of the item  $\chi_i$  and the opportunity cost  $\mathcal{E}$

$$h_i = c_i \mathcal{E} \quad (6.8)$$

Then, using (6.8) in (6.7)

$$\begin{aligned} \max_z \{ & \sum_{i=1}^T [\sum_{j=1}^U (n_{i,j} \prod_{m \in \Omega_j} (\Phi(z_m) + (1 - \Phi(z_m))w_m) p_i - \\ & - (\frac{Q_i}{2} + z_i \sigma_i \sqrt{L_i}) c_i \mathcal{E}] \} \end{aligned} \quad (6.9)$$

Note that  $z$  is not constrained. In order to satisfy the mean demand and have a safety stock, in fact we would need  $z > 0$ , but in this case we expect that for some items, it would be advantageous not to hold any inventory, or to hold inventory minor to the mean demand so the  $z < 0$  cases should be considered.

This way however, if

$$z_i < -\frac{Q_i}{2\sigma_i \sqrt{L_i}} \quad (6.10)$$

Then in equation (6.9) a negative holding cost would appear, that is to say that a “holding profit” would be computed. This is obviously not correct and thus (6.10) can be corrected as

$$\begin{aligned} \max_z \{ & \sum_{i=1}^T [\sum_{j=1}^U (n_{i,j} \prod_{m \in \Omega_j} (\Phi(z_m) + (1 - \Phi(z_m))w_m) p_i - \\ & - \max((\frac{Q_i}{2} + z_i \sigma_i \sqrt{L_i}) c_i \mathcal{E}, 0)] \} \end{aligned} \quad (6.11)$$

For the purpose of discussion, one could argue that a holding profit would make sense if backlogging wouldn't cause to actually lose sales (which is not considered in the (Q,R)

model) and if the company received the payments in advance. In that case, it would be advantageous for the company to have a backlog: the customer pays, the company does not have the item in stock and ships with delay, as a result the company earns the interest on the money the customer paid in advance. However, this makes no sense in reality because customers are not happy to wait and might not accept the estimated lead time (thus the order and the relative profit would be lost for the company) or might not buy from the company again in the future. Also, in many cases, as in the one considered here, it is not true that the customer pays in advance.

### **6.1.2 Implementation and feasibility**

The previously explained algorithm presents a significant drawback: the objective function is complex and requires a long time to be computed, especially when, as in our case, a large number of variables must be considered.

In this sense, some tests were carried out to evaluate how the algorithm scales as the number of variables and of orders increases.

The algorithm was implemented in Matlab and, as a first attempt, *fminunc* function was used. As shown below however, even before thinking about which solver should be used, one can understand that this formulation is simply not usable for the purpose of this work. In fact the time required to compute the objective function tends to be large and scales badly when the number of variables increases.

For the first experiments, the first 1000 lines of the orders database were considered. In it, 241 orders and 424 unique items were found. Note that every line represents one item in one order. In the other experiments, 5,000 and 10,000 lines were considered containing an increasing number of orders and unique items as reported in table 6-1 and in the figure 6-1. Note that in one year we could expect to have about 100,000 lines for the considered manufacturing area.

The solving machine was a Core 2 Duo 2.5Ghz based laptop which has a standard computational power for a new computer purchased in 2009, the computation times were measured using the “tic” and “toc” Matlab functions.

Table 6-1

**First Approach Objective Function complexity**

Lines	Orders	Items	Obj.func evaluation time [s]
1000	241	424	29.7365
5000	573	595	78.6685
10000	6623	1781	917.63

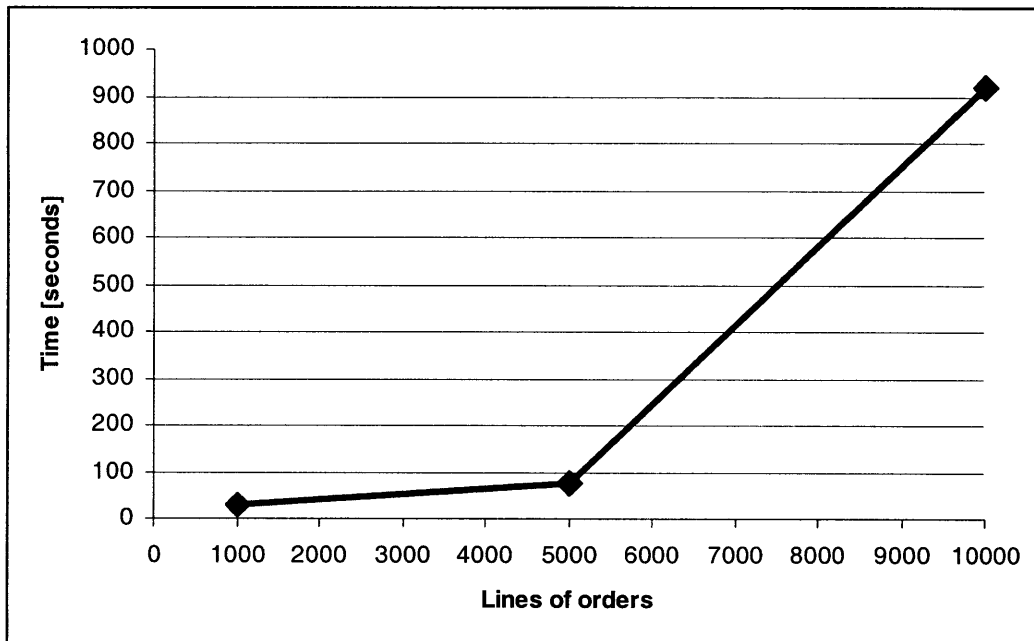


Figure 6-1

As one can see, the time required to evaluate the objective function grows quickly and is large already with 10,000 lines. In fact, even considering only 10,000 lines (instead of 100,000) and assuming the solver might find a solution with just 1,000 function calls (with a lot of variables it might require many more ), it would take 917,630 [s], equivalent to more than 10 days, to find a solution to the problem.

### **6.1.3 Conclusion**

The approach described in this chapter considers both the demand correlation and the customer impatience to determine an optimal reorder level for the (Q,R) policy. However the optimization problem which follows is too complex to be used with a large set of data and many variables. This complexity comes from two main reasons:

- All the computation happens inside the objective function. If, instead, some data could be pre-computed outside of the objective function, a lot of time could be saved.
- The approach tries to solve one single large problem with many variables. A set of small problems in place of a single large one would be significantly faster to solve.

## 6.2 Second Formulation

### 6.2.1 The Formulation

The optimization method described in 6.1 has the advantage of a large single optimization problem (accurate solution, possibility to add global constraints) but also its main disadvantage, that is to say the long computation time required to solve it. As seen, the previous method becomes unusable as the number of variables and of orders get larger and approaches the size of the problem we are interested in solving. Also, the previous method is not easily modifiable to take advantage of a smaller data set. The data presented in 6.1.2 in fact explores the possibility of a smaller problem by considering a reduced number of “data lines”. In reality however it is not possible to simplify the problem this way. Instead, it is possible and of practical interest to consider all the orders (then all the order lines) but determine the inventory policy for only some products. For example, as we did in our analysis, we can focus on a certain category of products and consider every item for the “possible profit” but only the ones belonging to that category for the purpose of determining the inventory levels. However an efficient exploitation of this simplification is not easy using the optimization formulation given in 6.1. Moreover, the approach proposed in 6.1 has a complex objective function because the information regarding the demand correlation is considered within it. The objective of this second formulation is to achieve the same result in a simpler and more computationally savvy way. This formulation ends up creating a single complex objective function which directly captures all the complexity of the problem but can be computed in a reasonable time. This formulation can be solved by a generic nonlinear solver as one of the solvers used by the *fminunc* Matlab function.

Let us start again from the consideration that we want to minimize the difference between a profit term and a cost term. Let us also define  $P(\text{sell } j)$  as the probability of the  $j$ -th order to be sold and  $\rho_j$  as the profit made by such order. The profit term is then a sum for every order of the profit made by that order multiplied by the probability of selling that order.

$$\sum_{j=1}^U P(\text{sell } j) \rho_j \quad (6.12)$$

As discussed in 6.1.1, considering that an order  $j$  can be shipped only if all the items  $m$  which are part of it ( $m \in \Omega_j$  where  $\Omega_j$  is the set of items requested in the order  $j$ ) are available and assuming that their availability is stochastically independent then (6.12) can be rewritten as:

$$\sum_{j=1}^U \prod_{m \in \Omega_j} [P(m \text{ is available})] \rho_j \quad (6.13)$$

Also, because the probability that an item  $m$  is available when an order arrives is what we defined as  $\alpha_m$ , (6.13) is equivalent to

$$\sum_{j=1}^U \prod_{m \in \Omega_j} [\alpha_m] \rho_j \quad (6.14)$$

Equation (6.14) assumes that every item must be in stock to allow an order to be shipped. However there is a percentage of clients willing to accept the production lead time if a part is not in stock, so considering (6.4) we can rewrite (6.14) as

$$\sum_{j=1}^U \prod_{m \in \Omega_j} [\alpha_m + (1 - \alpha_m) w_m] \rho_j \quad (6.15)$$

Finally, as shown in (5.10) assuming the demand to be normally distributed  $\alpha_i = \Phi(z_i)$

$$\sum_{j=1}^U \prod_{m \in \Omega_j} [\Phi(z_m) + (1 - \Phi(z_m)) w_m] \rho_j \quad (6.16)$$

Consistently with paragraph 6.1 we can then assume for each item  $i$  the holding cost to be proportional to its expected inventory level. So the total holding cost is:



$$\sum_{i=1}^T E(I_i) c_i \varepsilon \quad (6.17)$$

Where  $C_i$  is the unit cost for item  $i$  and  $\varepsilon$  is the opportunity cost for the company.

Considering equations (6.16) and (6.17) the objective function can be written as:

$$\max_z \left\{ \sum_{j=1}^U \prod_{m \in \Omega_j} [\Phi(z_m) + (1 - \Phi(z_m)) w_m] \rho_j - \sum_{i=1}^T E(I_i) c_i \varepsilon \right\} \quad (6.18)$$

To express the expected inventory level we can consider (5.14). However that expression can return negative values for  $z$  small enough. Because a “holding profit” does not make sense in this case, as discussed in 6.1 we will consider a holding cost only if  $E(I_i)$  is positive. Consequently, the optimization problem can be formulated as

$$\max_z \left\{ \sum_{j=1}^U \prod_{m \in \Omega_j} [\Phi(z_m) + (1 - \Phi(z_m)) w_m] \rho_j - \sum_{i=1}^T \max \left[ \frac{Q_i}{2} + z_i \sigma_i \sqrt{L_i}, 0 \right] c_i \varepsilon \right\} \quad (6.19)$$

Comparing equation (6.19) with (6.11) the key difference is that the double sum of equation (6.11) is no longer present here. Instead a single sum on the orders is necessary to compute the expected profit and a single sum on the items is necessary to compute the holding costs. Also, note that the term  $\rho_j$  is now pre-computed before starting to solve the optimization problem.

As a side note, the two separate sums are especially efficient when a dual core processor is used. In this case in fact the two sums can be computed in parallel by the microprocessor.

## 6.2.2 Gradient

Let us assume there are  $v$ , with  $v < T$ , items to be considered. The Gradient of the objective function is:

$$\begin{aligned} \nabla f &= \left( \frac{\partial}{\partial z_1} f, \dots, \frac{\partial}{\partial z_v} f \right) = \\ &= \left( \frac{\partial}{\partial z_1} (PROFIT\ TERM) + \frac{\partial}{\partial z_1} (HCTERM), \dots, \frac{\partial}{\partial z_v} PROFIT\ TERM + \frac{\partial}{\partial z_v} HCTERM \right) \end{aligned} \quad (6.20)$$

## DERIVATIVE OF PROFIT TERM

Recalling the expression (6.16) for the profit term, its partial derivative with respect to the  $k$ -th variable  $z_k$  is

$$\frac{\partial}{\partial z_k} \sum_{j=1}^U \prod_{m \in \Omega_j} [(\Phi(z_m) + (1 - \Phi(z_m))w_m)] \rho_j \quad (6.21)$$

In order to have a more compact formula let us define

$$\gamma(z_m) = \Phi(z_m) + (1 - \Phi(z_m))w_m \quad (6.22)$$

Considering the expression g.3 and the linearity of the partial derivative, (6.21) can be rewritten as

$$\sum_{j=1}^U \frac{\partial}{\partial z_k} \prod_{m \in \Omega_j} [\gamma(z_m)] \rho_j \quad (6.23)$$

Introducing the variable

$$\dim(\Omega_j) = \omega_j \quad (6.24)$$

And by applying the product rule for differentiation states and equation (6.23) becomes

$$\sum_{j=1}^U \left[ \left( \frac{\partial}{\partial z_k} \gamma(z_{m_1}) \dots \gamma(z_{m_{\omega_j}}) + \dots + \gamma(z_{m_1}) \dots \frac{\partial}{\partial z_k} \gamma(z_{m_{\omega_j}}) \right) \rho_j \right] \quad (6.25)$$

However note that

$$\begin{aligned} \frac{\partial}{\partial z_k} \gamma(z_{m_i}) &= 0 && \text{If } m_i \neq k \\ \frac{\partial}{\partial z_k} \gamma(z_{m_i}) &\neq 0 && \text{If } m_i = k \end{aligned} \quad (6.26)$$

More specifically, with reference to eq. (p.1.4) in Proof 6-1 at the end of this paragraph, (6.26) can be written as

$$\begin{aligned} \frac{\partial}{\partial z_k} \gamma(z_{m_i}) &= 0 && \text{If } m_i \neq k \\ \frac{\partial}{\partial z_k} \gamma(z_{m_i}) &= \varphi(z_{m_i})(1 - w_{m_i}) && \text{If } m_i = k \end{aligned} \quad (6.27)$$

Therefore using (6.27) in equation (6.25) we have that the derivative of the profit term is

$$\frac{\partial}{\partial z_k} PROFITTERM = \sum_{j=1}^U [\zeta(z_k) \rho_j] \quad (6.28)$$

where:

$$\left\{ \begin{array}{ll} m_i \in \Omega_j & \\ \zeta(z_k) = \gamma(z_{m_1}) \dots \frac{\partial}{\partial z_k} [\gamma(z_k)] \dots \gamma(z_{m_{\omega_j}}) & \text{If } k \in \Omega_j \\ \zeta(z_k) = 0 & \text{If } k \notin \Omega_j \end{array} \right.$$

## DERIVATIVES OF COST TERM

Recalling the expression (6.17) for the cost term, its partial derivative with respect to the  $k$ -th variable  $z_k$  is

$$\frac{\partial}{\partial z_k} \sum_{i=1}^T \left[ \max \left( \frac{Q_i}{2} + z_i \sigma_i \sqrt{L_i}, 0 \right) c_i \varepsilon \right] \quad (6.29)$$

Considering the linearity of the partial derivative, (6.29) can be rewritten as

$$\sum_{i=1}^T \frac{\partial}{\partial z_k} \left[ \max \left( \frac{Q_i}{2} + z_i \sigma_i \sqrt{L_i}, 0 \right) c_i \varepsilon \right] \quad (6.30)$$

Note that

$$\frac{\partial}{\partial z_k} \left[ \max \left( \frac{Q_i}{2} + z_i \sigma_i \sqrt{L_i}, 0 \right) c_i \varepsilon \right] = 0 \quad \text{If } i \neq k \quad (6.31)$$

Therefore equation (6.30) is equivalent to

$$\frac{d}{dz_k} \left[ \max \left( \frac{Q_k}{2} + z_k \sigma_k \sqrt{L_k}, 0 \right) c_k \varepsilon \right] \quad (6.32)$$

Recalling the following derivative

$$\frac{d}{dx} [\max(x, 0)] = \frac{1}{2} + \frac{\text{sign}(x)}{2} \quad (6.33)$$

Using equations (6.33) and (6.32) we find that the derivative of the cost term is

$$\frac{\sigma_k \sqrt{L_k} c_k \varepsilon}{2} \left[ 1 + \text{sign} \left( \frac{Q_k}{2} + z_k \sigma_k \sqrt{L_k} \right) \right] \quad (6.34)$$

## GRADIENT

In conclusion, using (6.27) and (6.34) the gradient is:

$$\nabla f = \left( \frac{\partial}{\partial z_1} F, \dots, \frac{\partial}{\partial z_k} F, \dots, \frac{\partial}{\partial z_v} F \right)$$

Where the  $k$ -th term is

$$\frac{\partial}{\partial z_k} F = \sum_{j=1}^U [\zeta(z_k) \rho_j] - \frac{\sigma_k \sqrt{L_k} c_k \varepsilon}{2} \left[ 1 + \text{sign} \left( \frac{Q_k}{2} + z_k \sigma_k \sqrt{L_k} \right) \right] \quad (6.35)$$

where:

$$\left\{ \begin{array}{ll} m_i \in \Omega_j & \\ \zeta(z_k) = \gamma(z_{m_1}) \dots \varphi(z_k) (1 - w_k) \dots \gamma(z_{m_{\omega_j}}) & \text{If } k \in \Omega_j \\ \zeta(z_k) = 0 & \text{If } k \notin \Omega_j \end{array} \right.$$

### Proof. 6-1

Having defined in (6.22)

$$\gamma(z_n) = \Phi(z_n) + (1 - \Phi(z_n))w_n \quad (\text{p.1.1})$$

Its derivative with respect to  $k$  is:

$$\frac{\partial}{\partial z_k} \gamma(z_k) = \frac{d}{dz_k} [\Phi(z_k) + (1 - \Phi(z_k))w_k] = (1 - w_k) \frac{d}{dz_k} \Phi(z_k) \quad (\text{p.1.2})$$

The first derivative of the normal cumulative function is

$$\frac{d}{dz_k} \Phi(z_k) = \frac{d}{dz_k} \int_{-\infty}^{z_k} \varphi(u) du \quad (\text{p.1.3})$$

And applying the fundamental theorem of calculus to (p.1.2)

$$\frac{d}{dz_k} \Phi(z_k) = \varphi(z_{n_i})(1 - w_{n_i}) \quad (\text{p.1.4})$$

Therefore, substituting (p.1.3) in (p.1.1) we obtain

$$\frac{\partial}{\partial z_k} \gamma(z_k) = \varphi(z_k)(1 - w_k) \quad (\text{p.1.5})$$

### 6.2.3 Hessian

The Hessian of the objective function is:

$$H(f) = \begin{bmatrix} \frac{\partial^2 F}{\partial z_1^2} & \dots & \frac{\partial^2 F}{\partial z_k \partial z_h} & \dots & \frac{\partial^2 F}{\partial z_1 \partial z_v} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \frac{\partial^2 F}{\partial z_k^2} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \frac{\partial^2 F}{\partial z_v \partial z_1} & \dots & \dots & \dots & \frac{\partial^2 F}{\partial z_v^2} \end{bmatrix}$$

Let us consider the mixed term and the diagonal ones separately.

### MIXED TERMS

Consider the generic mixed term  $\frac{\partial^2 F}{\partial z_k \partial z_h}$

With reference to expression (6.35), its partial derivative with respect to  $z_h$  is

$$\frac{\partial}{\partial z_h} \left\{ \sum_{j=1}^U [\zeta(z_k) \rho_j] - \frac{\sigma_k \sqrt{L_k} c_k \varepsilon}{2} \left[ 1 + \text{sign} \left( \frac{Q_k}{2} + z_k \sigma_k \sqrt{L_k} \right) \right] \right\} \quad (6.36)$$

Because only a part of (6.36) is function of  $z_h$  and considering the linearity of the partial derivation operator, we can rewrite it as

$$\sum_{j=1}^U \left[ \frac{\partial}{\partial z_h} \zeta(z_k) \rho_j \right] \quad (6.37)$$

Similarly to (6.23) we can apply the product rule for differentiation states and note that as in (6.26)



$$\begin{aligned}
\frac{\partial}{\partial z_k} \gamma(z_{m_i}) &= 0 && \text{If } m_i \neq h \\
\frac{\partial}{\partial z_k} \gamma(z_{m_i}) &= \varphi(z_{m_i})(1 - w_{m_i}) && \text{If } m_i = h
\end{aligned} \tag{6.38}$$

Therefore the considered mixed term is equal to

$$\frac{\partial^2 F}{\partial z_k \partial z_h} = \sum_{j=1}^U [\xi_M(z_k) \rho_j] \tag{6.39}$$

where:

$$\left\{ \begin{array}{ll}
m_i \in \Omega_j & \\
\xi_M(z_k) = \gamma(z_{m_1}) \dots \varphi(z_k)(1 - w_k) \dots & \\
\quad \dots \varphi(z_h)(1 - w_h) \dots \gamma(z_{m_{\omega_j}}) & \text{If } k \in \Omega_j \wedge h \in \Omega_j \\
\xi_M(z_k) = 0 & \text{Otherwise}
\end{array} \right.$$

## DIAGONAL TERMS

Consider now the generic term  $\frac{\partial^2 F}{\partial z_k^2}$

With reference to expression (6.35), its partial derivative with respect to  $z_k$  is

$$\frac{\partial}{\partial z_k} \left\{ \sum_{j=1}^U [\zeta(z_k) \rho_j] - \frac{\sigma_k \sqrt{L_k} c_k \varepsilon}{2} \left[ 1 + \text{sign} \left( \frac{Q_k}{2} + z_k \sigma_k \sqrt{L_k} \right) \right] \right\} \quad (6.40)$$

Considering that the derivative of the function sign is equal to zero

$$\frac{d}{dx} \text{sign}(x) = 0 \quad (6.41)$$

Equation (6.40) reduces to

$$\sum_{j=1}^U \left[ \frac{\partial}{\partial z_k} \zeta(z_k) \rho_j \right] \quad (6.42)$$

In this case equation (6.27) is still valid and we also know that the only terms function of  $z_k$  are of the type

$$\varphi(z_k)(1 - w_k) \quad (6.43)$$

The partial derivative of (6.43) is itself multiplied by  $z_k$

$$\frac{\partial}{\partial z_k} \varphi(z_k)(1 - w_k) = -z_k \varphi(z_k)(1 - w_k) \quad (6.44)$$

Therefore the  $k$ -th diagonal term is equal to

$$\frac{\partial^2 F}{\partial z_k^2} = \sum_{j=1}^U [\xi_D(z_k) \rho_j] \quad (6.45)$$

where:

$$\left\{ \begin{array}{ll} m_i \in \Omega_j & \\ \xi_D(z_k) = \gamma(z_{m_1}) \dots - z_k \varphi(z_k) (1 - w_k) \dots \gamma(z_{m_{w_j}}) & \text{If } k \in \Omega_j \\ \xi_D(z_k) = 0 & \text{If } k \notin \Omega_j \end{array} \right.$$

# 7 Decomposed optimization

## 7.1 Divide et impera

Although the formulation presented in 6.2 is usable, it requires some time for a computer to find a solution and problems might arise as the number of variables grows.

The objective of this approach is to achieve a similar result by breaking the problem in many easier sub problems, one for every variable. The reason why this is not directly feasible is because the correlation between the products makes the optimal service level of every item dependent on the others.

The proposed approach is then to use the function described in (5.21)

$$\max \sum_{i=1}^I E(S_i)p_i - h_i E(I_i)$$

in which a virtual profit, instead of the real profit is used. The virtual profit will account for the correlation between the products and will be computed in advance, before starting the optimization. As discussed in [27] the virtual profit is defined as

$$V_i = \sum_k N_{ik} p_k \tag{7.1}$$

- $i = 1, 2, \dots, T$
- $k = 1, 2, \dots, T$
- $N_{ik}$  is the number of times item  $i$  and  $k$  have been sold together where  $k$  can be equal to  $i$ , and  $N_{ii}$  simply indicates the number of times item  $i$  has been sold.

Note that  $N_{ik}$  can be zero.

- $p_k$  is the unit average profit made by item  $k$

## 7.2 The formulation

Let us consider the case of Otc orders first, that is to say every product can be shipped alone if the other products in the same orders are not available. Because every variable in 7.2 and 7.3 refers to Otc orders, let us suppress the superscript  $\mathcal{J}$  for the moment.

Considering the total profit  $P_i$  made by item  $i$  in OTC sales and the Type II service rate as defined in 5.2.3, the objective function can be written as:

$$\max_{\beta} \sum_{i=1}^T \beta_i P_i - h_i E(I_i) \quad (7.2)$$

Similarly to what is said for the first approach, because there is a percentage of customers available to accept the production lead time if a part is not in stock, the probability that a customer buy item  $i$  is the sum of the probabilities that “item  $i$  is in stock” and “the customer accepts to wait given that the customer is not in stock”. Therefore in place of  $\beta_i$  we can consider the term

$$\beta_i + (1 - \beta_i)w_i \quad (7.3)$$

Using (7.3) equation (7.2) becomes

$$\max_{\beta} \sum_{i=1}^T (\beta_i + (1 - \beta_i)w_i) P_i - h_i E(I_i) \quad (7.4)$$

If we use a (Q,R) policy then the average inventory level for each item is given by (5.14) and the objective function is:

$$\max_{\beta} \left\{ \sum_{i=1}^T [(\beta_i + (1 - \beta_i)w_i) P_i - (\frac{Q_i}{2} + z_i \sigma_i \sqrt{L_i}) h_i] \right\} \quad (7.5)$$

Or equivalently

$$\max_{\beta} \left\{ \sum_{i=1}^I [(\beta_i (1 - w_i) + w_i) P_i - (\frac{Q_i}{2} + z_i \sigma_i \sqrt{L_i}) h_i] \right\} \quad (7.6)$$

Because  $\beta_i$  and  $z_i$  are tied by (5.18) then (7.6) becomes

$$\max_z \left\{ \sum_{i=1}^I \left[ \left( \left( 1 - \frac{Plf(z_i)\sigma_i\sqrt{L_i}}{Q_i} \right) (1 - w_i) + w_i \right) P_i - \left( \frac{Q_i}{2} + z_i\sigma_i\sqrt{L_i} \right) h \right] \right\} \quad (7.7)$$

Finally, considering equation (6.8),

$$h_i = c_i \varepsilon$$

Then (7.7) can be rewritten as:

$$\max_z \left\{ \sum_{i=1}^I \left[ \left( \left( 1 - \frac{Plf(z_i)\sigma_i\sqrt{L_i}}{Q_i} \right) (1 - w_i) + w_i \right) P_i - \left( \frac{Q_i}{2} + z_i\sigma_i\sqrt{L_i} \right) c_i \varepsilon \right] \right\} \quad (7.8)$$

Note that in this formula, every parameter can be computed before running the optimization and can be stored in vectors. The only variables are the  $z_i$ .

More importantly, the objective function is given by the sum of many terms

$$\left( \left( 1 - \frac{Plf(z_i)\sigma_i\sqrt{L_i}}{Q_i} \right) (1 - w_i) + w_i \right) P - \left( \frac{Q_i}{2} + z_i\sigma_i\sqrt{L_i} \right) c_i \varepsilon \quad (7.9)$$

And each of them is independent one with the other

So the optimization problem to solve can be divided into  $v$  (the number of products for which we want to control the inventory) easier problems. In particular, for the generic  $k$ -th item, the problem to solve is

$$\max_{z_k} \left\{ \left( \left( 1 - \frac{Plf(z_k)\sigma_k\sqrt{L_k}}{Q_k} \right) (1 - w_k) + w_k \right) P_k - \left( \frac{Q_k}{2} + z_k\sigma_k\sqrt{L_k} \right) c_k \varepsilon \right\} \quad (7.10)$$

An important caveat with this formulation is that the expected inventory level

$$E(I) = \frac{Q_k}{2} + z_k \sigma_k \sqrt{L_k} \quad (7.11)$$

Might be negative. As discussed in chapter 6 this happens if

$$z_k < -\frac{Q_k}{2\sigma_k \sqrt{L_k}}$$

And as a consequence in equation (7.10) a negative holding cost would appear, that is to say that a “holding profit” would be computed. To avoid this, the problem can be re-formulated as:

$$\max_{z_k} \{F_k\}$$

$$F_k = \begin{cases} ((1 - \frac{Plf(z_k)\sigma_k \sqrt{L_k}}{Q_k})(1 - w_k) + w_k)P_k & \text{If } z_k < -\frac{Q_k}{2\sigma_k \sqrt{L_k}} \\ ((1 - \frac{Plf(z_k)\sigma_k \sqrt{L_k}}{Q_k})(1 - w_k) + w_k)P_k - (\frac{Q_k}{2} + z_k \sigma_k \sqrt{L_k})c_k \varepsilon & \text{If } z_k \geq -\frac{Q_k}{2\sigma_k \sqrt{L_k}} \end{cases} \quad (7.12)$$

Note that the two expressions for  $F_k$  coincide when

$$z_k = -\frac{Q_k}{2\sigma_k \sqrt{L_k}} \quad (7.13)$$

## 7.3 Analytical Solution

The optimization problem defined in (7.12) is constituted by the maximization of a function (defined by cases) of a single variable  $z_k$ . As shown here, an analytical solution to the problem can thus be found.

### 7.3.1 First derivative and stationary points

Let's consider the  $z_k < -\frac{Q_k}{2\sigma_k\sqrt{L_k}}$  case first

$$\text{If } z_k < -\frac{Q_k}{2\sigma_k\sqrt{L_k}}$$

By definition of  $F_k$

$$\frac{dF_k}{dz_k} = \frac{d}{dz_k} \left( \left(1 - \frac{Plf(z_k)\sigma_k\sqrt{L_k}}{Q_k}\right)(1 - w_k) + w_k \right) P_k \quad (7.2.1)$$

Which is equivalent to

$$-\frac{\sigma_k\sqrt{L_k}}{Q_k}(1 - w_k)P_k \frac{d}{dz_k} Plf(z_k) \quad (7.2.2)$$

Substituting the definition of Partial loss function given in (5.17), (7.2.2) becomes

$$-\frac{\sigma_k\sqrt{L_k}}{Q_k}(1 - w_k)P_k \frac{d}{dz_k} [\varphi(z_k) - z_k(1 - \Phi(z_k))] \quad (7.2.3)$$

Consider the following derivatives

$$\frac{d}{dz_k} [\varphi(z_k)] = -z_k\varphi(z_k) \quad (7.2.4)$$

$$\frac{d}{dz_k} [z_k\Phi(z_k)] = \Phi(z_k) + z_k\varphi(z_k) \quad (7.2.5)$$



Using (7.2.5) and (7.2.4) then (7.2.3) is equal to

$$-\frac{\sigma_k \sqrt{L_k}}{Q_k} (1 - w_k) P_k [-z_k \varphi(z_k) - 1 + \Phi(z_k) + z_k \varphi(z_k)] \quad (7.2.6)$$

And by simplifying (7.2.6) we get

$$-\frac{\sigma_k \sqrt{L_k}}{Q_k} (1 - w_k) P_k [-1 + \Phi(z_k)] \quad (7.2.7)$$

Now, knowing the first derivative of  $F_k$  we can look for singular points.

Equating the derivative to zero we have:

$$\frac{d}{dz_k} F_k = -\frac{\sigma_k \sqrt{L_k}}{Q_k} (1 - w_k) P_k [-1 + \Phi(z_k)] = 0 \quad (7.2.8)$$

Which is equivalent to require that

$$\Phi(z_k) = 1 \quad (7.2.9)$$

And equation (7.2.9) does not have a solution and therefore there are no singular points.

Also note that

$$\frac{d}{dz_k} F_k = -\frac{\sigma_k \sqrt{L_k}}{Q_k} (1 - w_k) P_k [-1 + \Phi(z_k)] > 0 \quad \forall z_k \quad (7.2.10)$$

Assuming that

$$\sigma_k > 0, L_k > 0, Q_k > 0, P_k > 0, w_k \in (0, 1) \quad (7.2.11)$$

And because

$$\Phi(z_k) > 0 \quad (7.2.12)$$

Equation (7.2.10) and (7.2.9) mean that the function is monotonically increasing and has no singular point. This is intuitive because, if there is no holding cost than

there is no reason to hold more and more inventory. By increasing  $z_k$  however

we reach the point in which it is no longer true that  $z_k < -\frac{Q_k}{2\sigma_k\sqrt{L_k}}$ .

Now let's consider the case  $z_k \geq -\frac{Q_k}{2\sigma_k\sqrt{L_k}}$

By definition of  $F_k$

$$\frac{dF_k}{dz_k} = \frac{d}{dz_k} \left[ \left( \left( 1 - \frac{Plf(z_k)\sigma_k\sqrt{L_k}}{Q_k} \right) (1 - w_k) + w_k \right) P_k - \left( \frac{Q_k}{2} + z_k\sigma_k\sqrt{L_k} \right) c_k \varepsilon \right] \quad (7.2.13)$$

Calculating the derivative and with reference to (7.2.8), (7.2.13) is equivalent to:

$$-\frac{\sigma_k\sqrt{L_k}}{Q_k} (1 - w_k) P_k [-1 + \Phi(z_k)] - \sigma_k\sqrt{L_k} c_k \varepsilon \quad (7.2.14)$$

Equation (7.2.14) is equal to zero if

$$z_k^* = \Phi^{-1} \left[ 1 - \frac{Q_k c_k \varepsilon}{(1 - w_k) P_k} \right] \quad (7.2.15)$$

So, with equation (7.2.15) one could find a stationary point which is a candidate to be the maximum.

However note that  $f(z) = \Phi^{-1}[x]$  is defined for  $x \in (0,1)$  therefore (7.2.15) is defined only if

$$1 - \frac{Q_k c_k \varepsilon}{(1 - w_k) P_k} > 0 \quad \wedge \quad 1 - \frac{Q_k c_k \varepsilon}{(1 - w_k) P_k} < 1 \quad (7.2.16)$$

Or equivalently if

$$Q_k c_k \varepsilon < (1 - w_k) P_k \quad \wedge \quad \frac{Q_k c_k \varepsilon}{(1 - w_k) P_k} > 0 \quad (7.2.17)$$

Under the assumptions (7.2.11) the second condition in (7.2.17) is always verified. The first condition instead, depending on which product is considered, might or might not be verified. We can conclude that if (7.2.17) is verified then a singular point exists and it is given by (7.2.15).

Also it is possible to show that

$$\frac{dF_k}{dz_k} < 0 \Leftrightarrow Q_k c_k \varepsilon \geq (1 - w_k) P_k \quad (7.2.18)$$

The consequence of these equations is shown in 7.3.3.

### 7.3.2 Second derivative

Because in (7.3.1) a stationary point is found, it is interesting to calculate the second derivative of  $F_k$  in such point.

First note that because the second derivative of the holding cost term is null

$$\frac{d^2}{dz_k^2} \left[ \left( \frac{Q_k}{2} + z_k \sigma_k \sqrt{L_k} \right) c_k \varepsilon \right] = 0$$

The second derivative of both cases  $z_k \geq -\frac{Q_k}{2\sigma_k \sqrt{L_k}}$  and  $z_k < -\frac{Q_k}{2\sigma_k \sqrt{L_k}}$  is the same .

expression of the first derivative in (7.2.8)

$$\frac{d^2 F_k}{dz_k^2} = \frac{d}{dz_k} \left[ -\frac{\sigma_k \sqrt{L_k}}{Q_k} (1 - w_k) P_k [-1 + \Phi(z_k)] \right] \quad (7.3.1)$$

Which is equal to

$$-\frac{\sigma_k \sqrt{L_k}}{Q_k} (1 - w_k) P_k [\varphi(z_k)] \quad (7.3.2)$$

Under the assumptions described in (7.2.11) and because

$$\varphi(z_k) : \mathbb{R} \mapsto \mathbb{R}^+ \quad (7.3.3)$$

Then

$$\frac{d^2 F_k}{dz_k^2} < 0 \quad (7.3.4)$$

Because (7.3.4) is valid in general it is also valid in the stationary point and therefore

$z_k^*$  defined in (7.2.15) is a maximum.

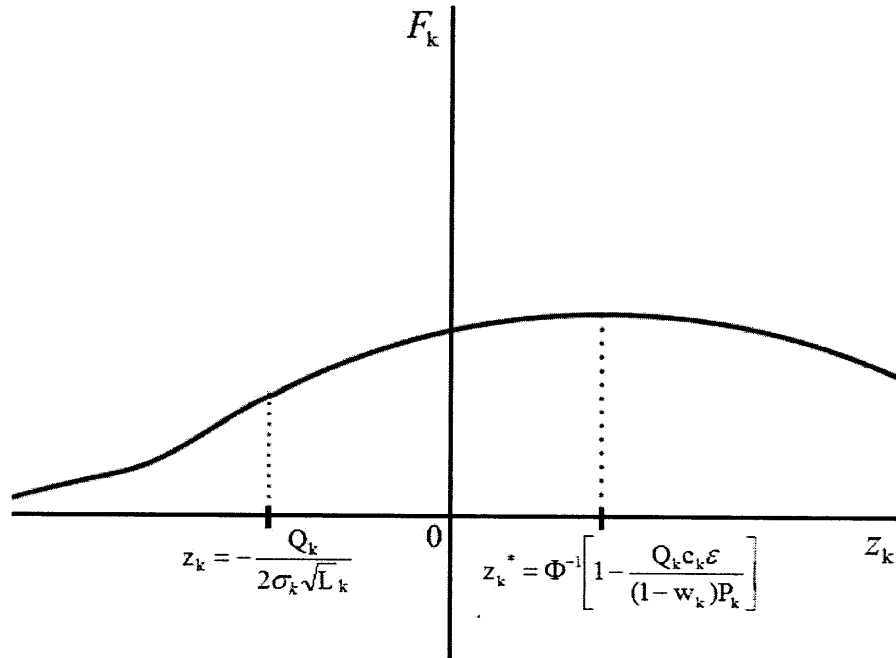
### 7.3.3 Study of the objective function

With reference to condition (7.2.15), for the  $k$ -th product two cases are possible:

1. If  $Q_k c_k \varepsilon < (1 - w_k) P_k$

$$\left\{ \begin{array}{l} \text{When } z_k < -\frac{Q_k}{2\sigma_k \sqrt{L_k}} \Rightarrow \frac{dF_k}{dz_k} > 0 \\ \text{When } z_k \geq -\frac{Q_k}{2\sigma_k \sqrt{L_k}} \Rightarrow \frac{dF_k}{dz_k} \text{ changes sign as a function of } z_k \end{array} \right.$$

Then the objective function is shaped as



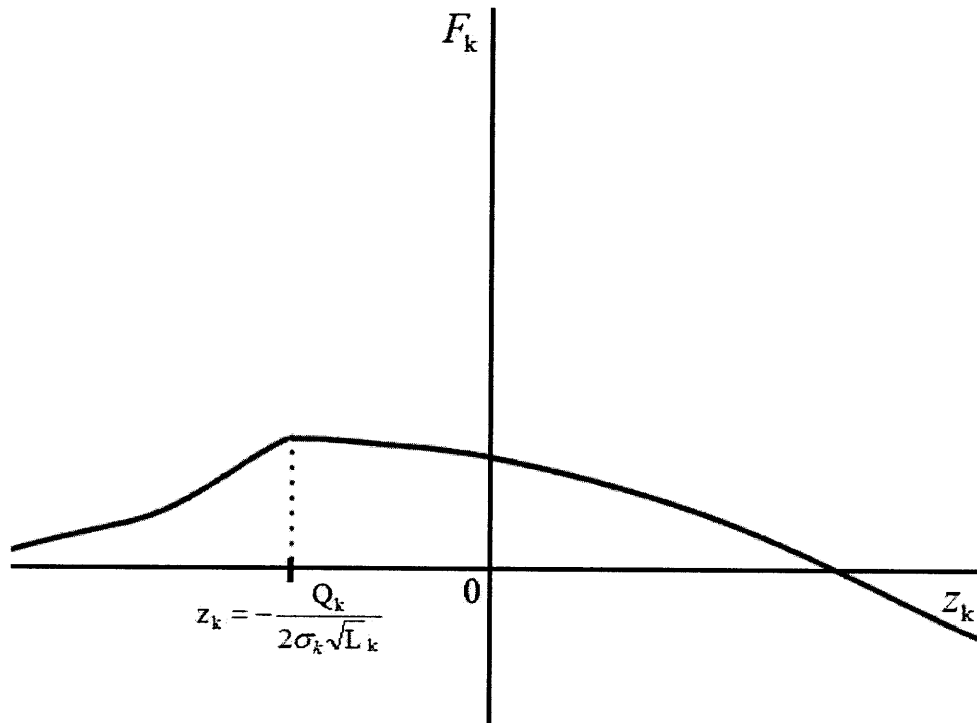
In this case the optimum is given by the maximum

$$z_k = \Phi^{-1} \left[ 1 - \frac{Q_k c_k \varepsilon}{(1 - w_k) P_k} \right]$$

2. If  $Q_k c_k \varepsilon \geq (1 - w_k) P_k$

$$\left\{ \begin{array}{l} \text{When } z_k < -\frac{Q_k}{2\sigma_k \sqrt{L_k}} \Rightarrow \frac{dF_k}{dz_k} > 0 \\ \text{When } z_k \geq -\frac{Q_k}{2\sigma_k \sqrt{L_k}} \Rightarrow \frac{dF_k}{dz_k} < 0 \end{array} \right.$$

Then the objective function is shaped as:



In this case the optimum is given by  $z_k = -\frac{Q_k}{2\sigma_k \sqrt{L_k}}$ , that is to say by the value that makes

the average inventory level equal to zero. The item then, should not be held in inventory and should be classified as Make to Order.

### 7.3.4 The policy

In conclusion the resulting policy for the  $k$ -th product is as follows.

- If  $Q_k c_k \varepsilon \geq (1 - w_k) P_k$

The product will not be held in inventory  $\rightarrow$  Make To Order Product

- If  $Q_k c_k \varepsilon < (1 - w_k) P_k$

The product will be held in inventory  $\rightarrow$  Make To Stock Product

The safety factor is chosen as:

$$z_k = \Phi^{-1} \left[ 1 - \frac{Q_k c_k \varepsilon}{(1 - w_k) P_k} \right]$$

The reorder level  $R$  is set as:

$$R_k = \mu_k L_k + z_k \sigma_k \sqrt{L_k}$$

An interpretation of the policy is: **if the lost profit that would arise in case the part is MTO is greater than the cost of holding  $Q_k$  parts in stock, then the part should be hold in stock. Otherwise, the part should be produced as a MTO.**

### 7.3.5 Notes about implementation

The policy described in 7.3.4 is based on the assumptions

$$\sigma_k > 0, L_k > 0, Q_k > 0, P_k > 0, w_k \in (0,1)$$

We expect these assumptions to be verified in the vast majority of cases, but sometimes they might not be.

The assumptions  $\sigma_k > 0$  might not be verified if the demand is absolutely constant. In this case there is no need to have a safety stock and the reorder level should be simply set at

$$R_k = \mu_k L_k \tag{7.5.1}$$

The assumption  $P_k > 0$  might not be respected if the part was never sold (the profit is null) or is sold at loss. In the first case the part should be set as MTO and in the second case a warning should be given to the user.

Moreover, all the assumptions, in a real world use, might then be not verified because of faulty data. In this case the user will be warned and a solution for that particular item will not be computed.

To account for these problems, the policy has been implemented as indicated described in 7.3.6



### 7.3.6 Implemented policy for Otc

Reintroducing from now on the superscripts  $\mathcal{G}$  and  $\zeta$  to differentiate the variables relative to Otc and Systems orders, the implemented policy for Otc is

- If  $\sigma_k^{\mathcal{G}} < 0 \vee L_k \leq 0 \vee Q_k \leq 0 \vee P_k < 0 \vee w_k \notin (0,1)$ 
  - Give a warning
- Else if  $P_k = 0$ 
  - Make To Order Product
- Else if  $\sigma_k^{\mathcal{G}} = 0$ 
  - Make To Stock Product
 
$$R_k = \mu_k L_k$$
- Else
  - If  $Q_k c_k \varepsilon \geq (1 - w_k) P_k$   
Make To Order Product
  - If  $Q_k c_k \varepsilon < (1 - w_k) P_k$   
Make To Stock Product

$$z_k^{\mathcal{G}} = \Phi^{-1} \left[ 1 - \frac{Q_k c_k \varepsilon}{(1 - w_k) P_k} \right], \quad R_k^{\mathcal{G}} = \mu_k^{\mathcal{G}} L_k + z_k^{\mathcal{G}} \sigma_k^{\mathcal{G}} \sqrt{L_k}$$

## 7.4 Systems and Over the Counter Orders

### 7.4.1 Proposed approach

In chapter 7.3 a proposed policy for each product is derived assuming each product can be shipped alone if the others with which it is ordered are not available. However, as previously discussed, this assumption is not valid for Systems orders. Our approach to face this issue, in the divide et impera spirit, is to consider the Otc and System orders separately. Let us consider the generic  $k$ -th product. The sequence of operations is as follow:

1. The reorder quantity  $Q_k$  is determined considering the total demand (Otc plus Systems) as described in 5.3.
2. Considering the total profit  $P_k$  made by the item in Otc orders and its demand mean  $\mu_k^g$  and standard deviation  $\sigma_k^g$  in Otc orders a reorder level  $R_k^g$  is determined as defined in 7.3.6
3. Considering the virtual profit  $V_k$  made by the item in Systems orders (in place of  $P_k$ ) and its demand mean  $\mu_k^s$  and standard deviation  $\sigma_k^s$  in Systems orders a reorder level  $R_k^s$  is determined as defined in 7.4.3.
4. The “partial” reorder levels  $R_k^s$  and  $R_k^g$  are summed up and rounded to an integer value following a heuristic rule as the one described in 7.4.4 This way a final value  $R$  is determined.

A discussion of this approach can be found in the next two paragraphs.

### 7.4.2 Discussion of Systems orders case

As presented in 7.4.1, in order to determine the reorder level systems orders are treated similarly to Otc orders but the total virtual profit  $V_k$  is used in place of the total individual profit  $P_k$ . To explain this choice let us refer to the expression of the objective function for Otc orders

$$\max_{z_k} \left\{ \left( \left( 1 - \frac{Plf(z_k)\sigma_k^g\sqrt{L_k}}{Q_k} \right) (1 - w_k) + w_k \right) P_k - \left( \frac{Q_k}{2} + z_k\sigma_k^g\sqrt{L_k} \right) c_k \varepsilon \right\} \quad (7.4.1)$$

In (7.4.1) the role of the term

$$\left[ \left( 1 - \frac{Plf(z_k)\sigma_k^g\sqrt{L_k}}{Q_k} \right) (1 - w_k) + w_k \right] P_k \quad (7.4.2)$$

Is to evaluate how much profit would be lost with the current safety factor  $Z_k$ . Because in an Otc order we assume that product  $k$  can be shipped regardless of the availability of other products in the same order it make sense to multiply the probability of selling it

$$\left[ \left( 1 - \frac{Plf(z_k)\sigma_k^g\sqrt{L_k}}{Q_k} \right) (1 - w_k) + w_k \right] \quad (7.4.3)$$

by the total profit realized by such item  $P_k$ .

In Systems orders however each part could prevent an entire order to be sold. For this reason we try to estimate the profit that would be lost with the current safety stock  $Z_k$  by multiplying the probability (7.4.3) with its virtual profit  $V_k$ , that is to say with the total profit made by the orders in which product  $k$  is sold. The objective function is then

$$\max_{z_k} \left\{ \left( \left( 1 - \frac{Plf(z_k)\sigma_k^\zeta \sqrt{L_k}}{Q_k} \right) (1 - w_k) + w_k \right) V_k - \left( \frac{Q_k}{2} + z_k \sigma_k \sqrt{L_k} \right) c_k \varepsilon \right\} \quad (7.4.5)$$

### 7.4.3 Implemented policy for Systems

The resulting policy considering Systems orders is, similarly to 7.3.6

- If  $\sigma_k^\zeta < 0 \vee L_k \leq 0 \vee Q_k \leq 0 \vee V_k < 0 \vee w_k \notin (0,1)$ 
  - Give a warning
- Else if  $V_k = 0$ 
  - Make To Order Product
- Else if  $\sigma_k^\zeta = 0$ 
  - Make To Stock Product
 
$$R_k^\zeta = \mu_k^\zeta L_k$$
- Else
  - If  $Q_k c_k \varepsilon \geq (1 - w_k) P_k$   
Make To Order Product
  - If  $Q_k c_k \varepsilon < (1 - w_k) P_k$   
Make To Stock Product

$$z_k^\zeta = \Phi^{-1} \left[ 1 - \frac{Q_k c_k \varepsilon}{(1 - w_k) P_k} \right], \quad R_k^\zeta = \mu_k^\zeta L_k + z_k^\zeta \sigma_k^\zeta \sqrt{L_k}$$

### 7.4.3 Discussion of sum rules

Once  $R_k^{\theta}$  and  $R_k^{\varsigma}$  are determined these two quantities must be considered together to obtain a total  $R$ . Determining the reorder level as:

$$R_k = R_k^{\theta} + R_k^{\varsigma} \quad (7.5.1)$$

is a conservative choice as it assumes that the demand for the  $k$ -th product in Otc orders is independent from its demand in Systems orders. Such assumption has not been tested but there is no evident reason why the two demands should be dependent. One could argue that customers who buy Systems will probably later buy spare parts as Otc but in the majority of cases this happen after a period of time larger than the one considered here. Therefore the assumption of independence is conservative but is expected not to be too far from reality. As a consequence, we expect (7.5.1) to give a conservative but not excessively large estimate of the reorder level needed.

Note that equation (7.5.1) cannot be used if the resulting policy for Otc or Systems orders is Make To Order as either  $R_k^{\theta}$  or  $R_k^{\varsigma}$  is undefined. Moreover, the reorder level to be used in practice must be an integer number so the results of (7.5.1) must be somehow rounded. A few heuristic rules were tested to face this issue and they resulted in minor differences in the final policy.

Object of future work can be to research the best rule to sum up the results of the Otc and Systems analysis. The proposed heuristic rule used in this work is described in 7.4.4.

#### 7.4.4 Proposed sum rule

The rule used to sum up the results of the Otc and Systems analysis in this work is:

- If ((Systems  $\rightarrow$  MTO) AND (Otc  $\rightarrow$  MTO))

MTO product

- Else

- If (Systems  $\rightarrow$  MTO)

MTS product with  $R'_k = R_k^{\mathcal{J}}$

- If (Otc  $\rightarrow$  MTO)

MTS product with  $R'_k = R_k^{\mathcal{C}}$

- Else

MTS product with  $R'_k = R_k^{\mathcal{J}} + R_k^{\mathcal{C}}$

- $R_k = \text{round}(R'_k)$

# 8 Comparison of the approaches

## 8.1 Quality of solutions

A first approach to determine the inventory levels is presented in 6.1. As discussed in that chapter, using such approach requires solving a large (with  $v$  variables if  $v$  is the number of items considered for inventory control) optimization problem. In chapter 7 instead an alternative approach is presented which decompose the problem into  $v$  simple optimization problems for which an analytical solution was found in 7.3. This approach also separates the computation for each item into two smaller ones: a subproblem for Otc orders and a subproblem for Systems orders. The advantages of this approach in comparison to the first in term of complexity are outlined in 8.2, but firstly it is important to determine how good the levels determined are. For this reason we tested the levels, simulating what would have happened in the same years (2007 and 2008) used to determine those levels. In other words the purpose of this test is not to understand how well the levels might perform in the future, but rather how well the optimization technique used the input data to determine the levels. The results comparing the global optimization as described in 6.2 and the decomposed optimization are shown in Figure 8-1. The size of inventory is measured in months of demands. For a definition of Months On Hand check paragraph 11.1

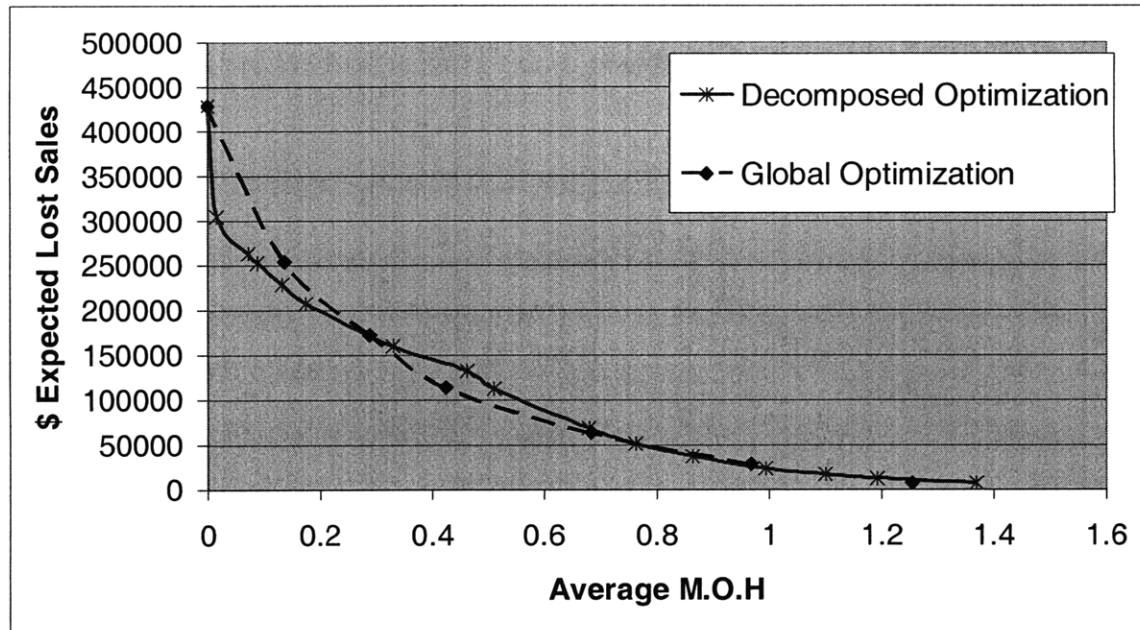


Figure 8-1

As one can see the two curves are rather close one to another. This means that both approaches manage to find good solutions to the tradeoff between inventory on hand and lost sales. The solution found with the first and more complex approach tends to be better but not in a significant way. Also, there is one point in which a worse solution is found by the global optimization possibly due to a local minimum found by the solver used. Figure 8-1 also shows that the two curves become very similar when the inventory on hand is larger than 0.8 MOH. It is indeed intuitive that as the number of items held in inventory grows it makes less difference how the mix of items was determined and how good it was. This also means that if a solution with an average inventory larger than 0.8 MOH is chosen it make a very small difference which approach is used to determine it.

## 8.2 Computational comparison

The reason why the approach presented in chapter 7 is attractive is its computational speed. For instance, if  $v=80$  products are considered and 2 years of data are analyzed the decomposed optimization can be computed in about 3 seconds (excluding the data import, skimming and analysis phase which needs about 20 minutes). The global



optimization instead (again excluding the data import and skimming phase) was observed to take about 5 hours to complete on the computer described in 6.1. This number is only a rough measure as every set of variables cause the solver to converge in a different number of iterations and thus in a different amount of time. Moreover, one should note that this number might probably be reduced with an appropriate choice of the starting value of the vector  $z$ . In the tests executed the initial point was simply a null vector but, analyzing the optimal  $z$  in various cases, one could probably build find an initial  $z$  which reduces the number of iterations. Anyway the advantage of using the decomposed approach is still evident, especially considering that, if  $v$  gets larger, the computation time required by the decomposed approach increases linearly while the one required by the global approach is expected to scale in a worse way.

### **8.3 Implementation issues**

From an implementation point of view the two approaches are almost equivalent. Of course, because the second approach does not require the use of a solver there are no constraints in term of programming language and environment. In other words, using such approach is easier to abandon Matlab and use another programming language as C or Visual Basic. However none of the approaches can be used with the sole aid of a spreadsheet as a large set data must be considered (more than Excel 2003 can handle for example), and more importantly many calculation must be performed to skim the data, determine the demand means and standard deviation for every item etc.

### **8.4 Conclusions**

The results of the previous paragraphs show that the use of the second approach is advisable as it is quick and produce good results. Possibly more important than the measured quickness is the fact that its complexity is linear with  $v$ , differently than the first approach. This assures that the decomposed approach could be used also for a very

large number of products. By showing that the approach presented in 7 can be used we showed that the inventory levels problem can be faced by:

- Decomposing the optimization problem into  $v$  small problems
- Further decomposing the problem for each item into two separate problems for Otc and Systems orders
- Finding an analytical solution for those problems
- Use the Virtual Profit to account for demand correlation and capability of each item to block the orders in which it is sold

# 9 Adapt to future market conditions

## 9.1 Demand shift

The solution proposed in the previous chapters assumes a stationary demand. To avoid making such an assumption a possible approach is to calculate the parameters  $R$  and  $Q$  using the forecasts for the future demand. For example, something similar is proposed in 32. In our case however this approach was not chosen because forecasts for each and every product were not available. Also, because of the huge number of products available in Instron catalog and of the different industries which use Instron products, it would be time consuming and difficult to make such forecasts. Excluded this possibility, a problem with changes in future demand still exist. This problem was evident from our simulations as discussed in the next paragraph.

### 9.1.1 The demand is not stationary

The effects of assuming stationary demand when there is, for example for macroeconomic reasons, a general mean shift, are critical and can undermine the performance of the policy. Figure 9-1 gives us a sample of this. Assume that a certain policy has been selected with an average of 1.2 MOH of inventory. If the following year the demand increased of up to 30% the average inventory would slightly decrease but there would be a higher stock out probability and therefore a higher number of lost sales (see [27] for a more in-depth analysis). If instead the demand decreased the average inventory level would quickly go up to a possibly unacceptable level. In figure 9-1 it is shown that a decrease of 30% can cause the size of the inventory to more than double. This is of course due to the way MOH is defined but still shows that it is important to consider an overall mean shift when determining the inventory control parameters.

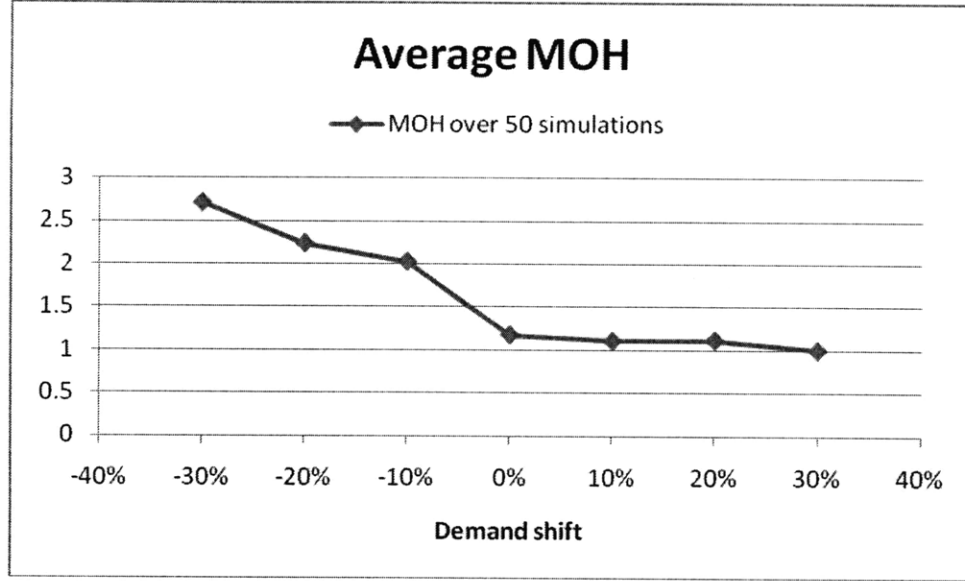


Figure 9-1

### 9.1.2 Proposed solution: a shift factor $\delta$

In order to face the problem outlined in 9.1.1 we introduce a shift factor  $\delta$ . This approach was chosen because it is:

- Simple for the user to identify  $\delta$ , possibly agreed throughout the whole company
- Simple for the user to input it to the proposed tool, as opposed to a product-by-product forecast (which might be still used, see 9.2)
- Effective in adjusting the inventory level so that the expected MOH is acceptable for the company

The factor  $\delta$  is used in the pre-processing phase and modifies the purchased quantities in the orders. This way for the  $k$ -th product also the variables  $\mu_k$ ,  $\sigma_k$ ,  $z_k$ ,  $\rho_k$ ,  $P_k$ ,  $V_k$ ,  $Q_k$ ,  $R_k$  are computed accordingly.

## 9.2 Introduction of new products

Another issue which might arise in the practical use of the proposed tool is that some item which existed in the past are going to be replaced in the future. For example, product A and B that were sold in the period used to determine the inventory policy might be substituted in the future by product C. In this case a solution might be to use the computed  $R$  and  $Q$  for item A and B to obtain the ones for product C. However sum up the values of  $R$  and  $Q$  is in general incorrect. Also, it might be that the product C does have a different cost or a different price than product A and B and this would affect value of  $z$  that should be used to determine  $R$ . For this reason, similarly to what proposed in 9.1 a direct substitution in the pre-processing phase is proposed. This way in every order in which A or B appear they will be replaced by C and the profit, the cost and the purchased quantity will be modified as well. Again, the direct modification of the order list allows all the variables used in the optimization and in the  $R$  and  $Q$  computation to be considered in the correct way.

Note that this tool might also been used to specify to the software specific forecast. For example, suppose the company decides to cut the price of item F by half and as a result expect the sales to double. In this case it can be inputted to the tool that item F will be substituted with item F with a 200% probability and the cost of the new product is the half of the one being substituted.

# 10. Results and discussion

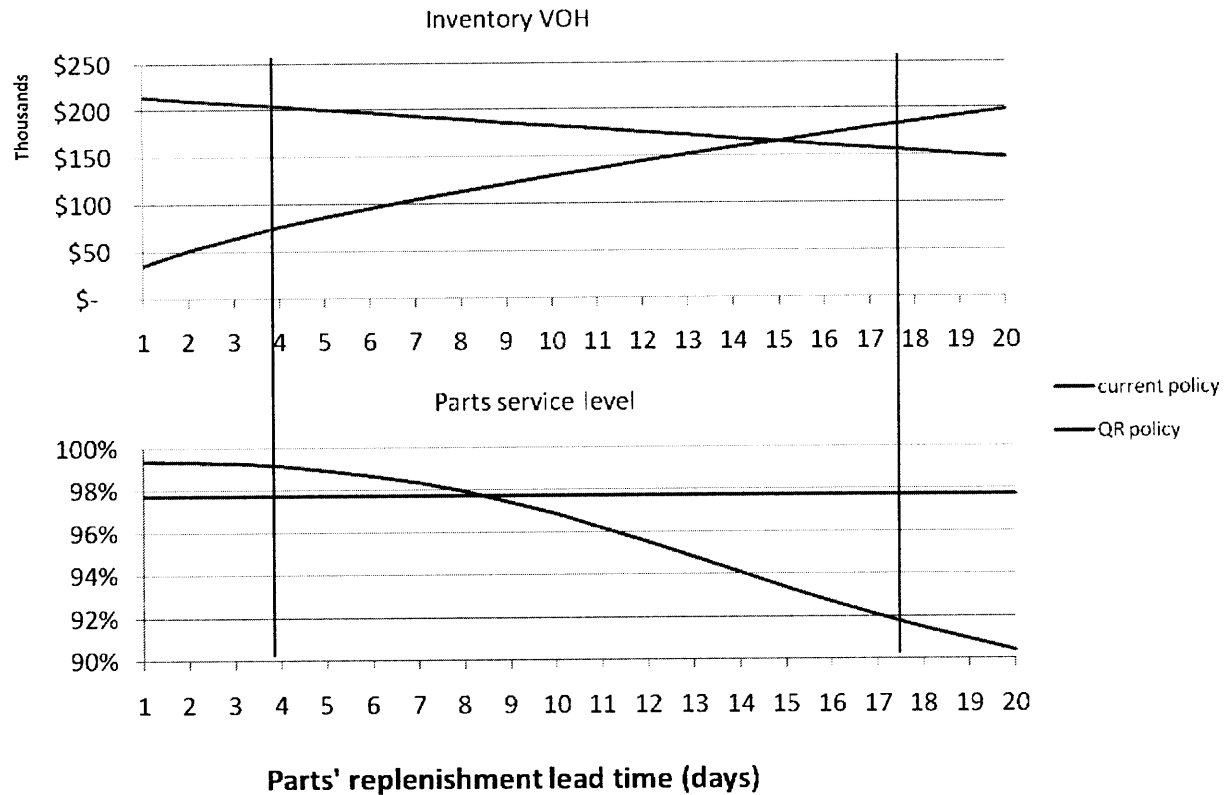
## 10.1 Raw materials inventory

As introduced in Chapter 4, purpose of the project was also to provide a raw materials inventory control policy supporting the finished goods inventory. The current policy is value-based: the parts are classified by financial value to the company (classes A, B, C and D) and the reorder quantities and levels only depend on the class. A Q,r policy with fixed service levels is proposed; the results are here summarized and discussed.

### 10.1.1 Results

In order to implement the Q,r policy for the raw materials inventory, some information is necessary. In particular, knowing the replenishment lead times negotiated with the supplier is fundamental. In this paragraph, the results of the Q,r policy are presented by comparison with the current value-based control policy. Firstly, the importance of the lead time is shown through a parametric comparison; then, the two policies are evaluated with the best current estimate of the lead times.

Figure 10-1 shows the difference that could be made by having more accurate information about the lead times. The graph on the top shows the expected inventory value on hand, while the graph below shows the average service levels. For the sole purpose of showing the differences as the lead times vary, the graphs are based on the assumption that the lead time is the same, and constant, for all the parts. The blue lines represent the current value-based policy, which does not consider lead times or the demand variability. The red lines correspond to the Q,r policy, implemented using also the lead times and variability information. Two examples are highlighted with vertical lines: a lead time of 4 days and a lead time of 18 days.



**Figure 10-1** Service level and Inventory VOH vs. lead time

If the suppliers ship more quickly than expected, and the lead time is shorter, both policies have high service levels because the shelves are replenished quickly. Being designed upon the shorter lead times, however, the Q,r policy manages to accomplish high service levels with low inventories. In the first example, indeed, the value on hand is reduced by three times.

On the other hand, if the lead times are longer, the only way to achieve high service levels is to have higher inventories. Thus, the proposed policy suggests inventory levels that are comparable or even higher than the current ones. The Q,r policy, on one hand, uses the information about lead times in order to maintain high service levels; the current policy, on the contrary, does not consider them, causing a significant percentage of lost orders (orders meaning grips to be assembled), as shown in the second example.

An estimate of the actual supplier replenishment lead times is obtained by talking with the purchasing department and described by Palano [26]. In this case, the lead times are different for each part. Table 10-1 shows a comparison of the results obtainable with the two policies based on this estimate. Moreover it provides an estimate of the savings that would be achieved by agreeing on shorter lead times with the suppliers.

Method	Average inventory VOH	Parts service level
ABCD - Division by value	\$179,731	93.2%
Q,r – Knowing and using the lead times	\$126,299 (-30%)	97.7%

**Table 10-1** Comparison of raw materials inventory control policies

As table 10-1 shows, only as a result of improving the accuracy of lead times, the Q,r policy would allow achieving high service levels at the same time cutting the costs by 30%. If, in addition, the purchasers obtain agreements for shorter lead times for the most valuable parts, the costs would further decrease.

### **10.1.2 Discussion**

Based on the analysis proposed by Palano [26] and on the results here described, the current inventory policy, which is value-based and does not consider lead time and demand variability, can result in irregular inventory distribution, lower service levels and higher inventory value on hand. A simple Q,r policy is proposed, which gives better and more regular results.

In designing and optimizing the finished goods inventory control, the assumption that all the raw materials are always available is made. The designed Q,r policy achieves service



levels of about 98% for each part. Thus, the above mentioned assumption can be still considered valid.

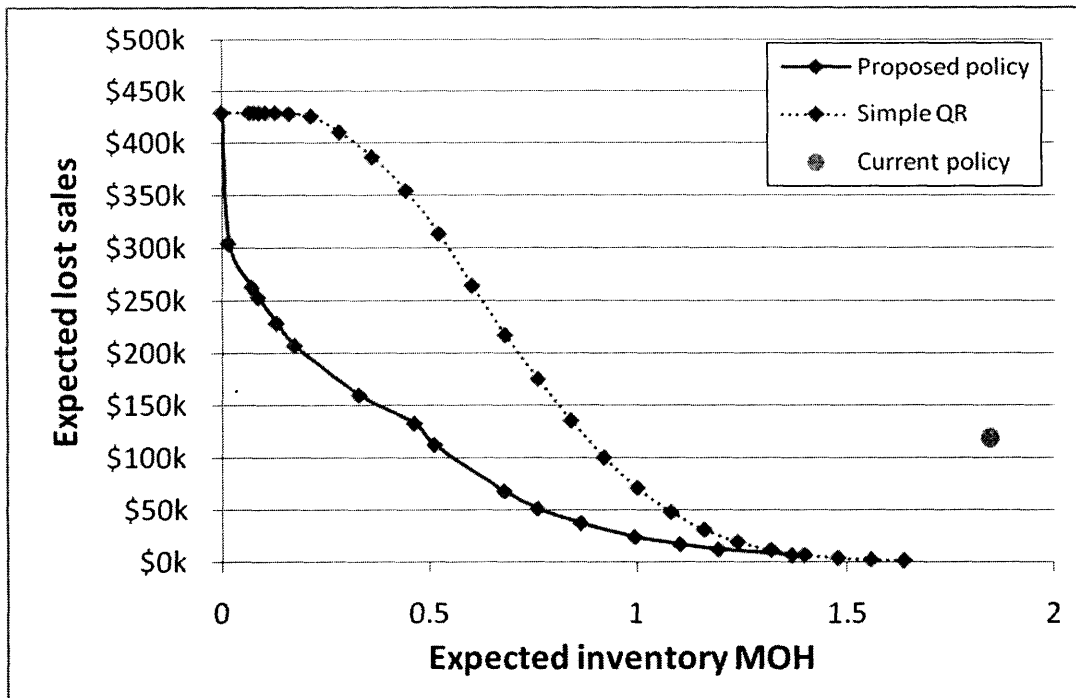
However, in order to implement the Q,r policy, the replenishment lead times are necessary. As a general consideration, the lead times are necessary to make sure that the service levels are high without wasting inventory. Thus, the lead times of every part should be tracked in the way described in section 11.1, and accurate information should be kept on the company databases. In addition, if the suppliers are flexible on the lead times, the Excel spreadsheets can be used in the decision process to determine the correct tradeoff between lead times and inventory value on hand.

## 10.2 Finished goods inventory

The policy proposed shows potential for a significant improvement in inventory control. Figure 10-2 shows a comparison between the proposed policy, a simple Q,r policy and the values of  $Q$  and  $R$  currently in use. Note that the term “simple Q,r” refers to a Q,r policy with an equal safety factor  $z$  for all the products. The figure shows the expected lost sales, due to products unavailability, versus the total expected inventory held. The amount of inventory held is measured in months on hand (MOH):

$$E[I]_{MOH} = \frac{\text{expected inventory value on hand}}{\text{average monthly demand}} = \frac{\sum_i c_i E[I_i]}{\sum_i c_i \mu_i} \quad (10.1)$$

Where  $c_i$  is the unit cost of part  $i$ ,  $E[I_i]$  is its expected inventory level and  $\mu_i$  is its average monthly demand.

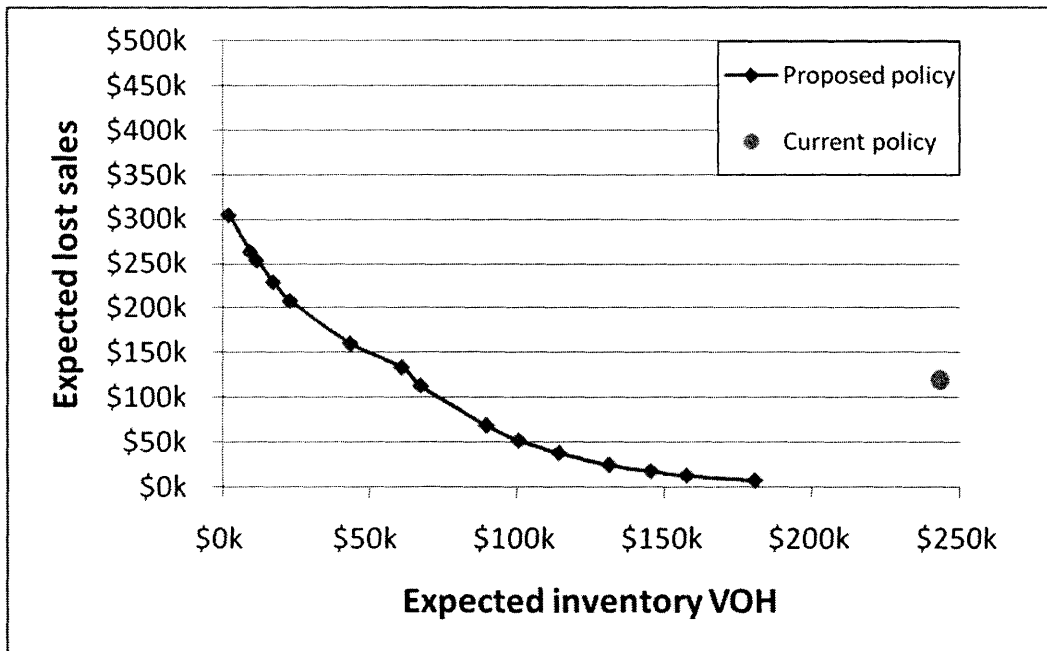


**Figure 10-2** Expected lost sales vs. Inventory MOH

As figure 10-2 shows, the proposed policy outperforms both the simple Q,r and the current policy. In particular, at the same level of expected loss sales given by the current policy, the proposed policy allows reducing the inventory from about 1.8 MOH to 0.5 MOH. From another point of view, with the amount of inventory currently held, the proposed policy would allow reducing the expected lost sales from about \$120,000 per year to nearly zero.

In addition, Figure 10-2 shows that the proposed policy outperforms the simple Q,r policy. As one might expect, the difference increases as the size of the inventory gets smaller, while it decreases as larger inventory is considered. As a limit case, the value of lost sales achieved by the simple Q,r with 0.15 MOH is the same that would be obtained by a complete make to order (MTO) policy. With the proposed policy, instead, 0.15 MOH of inventory can halve the expected loss as compared to an MTO policy.

Figure 10-3 shows the expected lost sales value versus the value of the inventory on hand. As one can see from the graph, if a solution with 1.2 MOH is chosen (the penultimate point on the purple line) the inventory could be reduced from \$240,000 to \$157,000.



**Figure 10-3** Expected lost sales vs. Inventory VOH

Considering the trade-off between size of inventory and expected loss sales, a good compromise is a solution with an expected inventory of 1.2 MOH. This allows both reducing the amount of inventory and the expected loss sales. Moreover, a preliminary analysis of the maximum inventory levels shows that, with this solution, it is unlikely that the inventory levels measured at the end of one month will go above 2 MOH (considering the monthly demand variability). Table 10-3 shows a comparison between the proposed solution (with 1.2 MOH) and the current policy.

	Current Policy	Proposed Policy
Average value of Lost orders	\$119,391	\$12,453
Expected Inventory (MOH)	1.85	1.19
Expected Inventory (VOH)	\$243,481	\$157,411

**Table 10-3** Current policy vs. 1.2 MOH solution

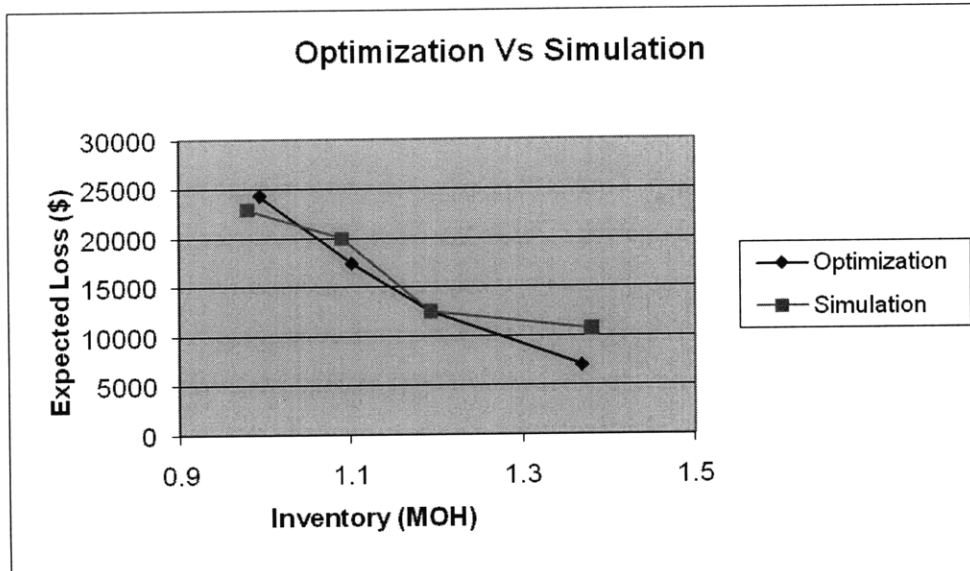
## 10.3 Simulation

The aim of simulation is to validate the results of the optimization module and to test the robustness of the proposed policy. The simulation also helps to determine the advantage of considering correlation between the demands of items sold in systems as compared to neglecting them in the analysis as explained by Palano [26]. The simulation estimates the following performance measures: number of orders lost, their value, months on hand of inventory for every month simulated and dollar value of inventory for each simulated day.

### 10.3.1 Validation

The optimization model provides the right mix of products that should be available on the floor. To validate these results, the levels were simulated 50 times over two years, 2007 and 2008, and then compared with the projected results from the optimization.

Figure 10-5 shows the losses made for different optimized inventory levels as predicted from the optimization and the simulation, versus the inventory months on hand.



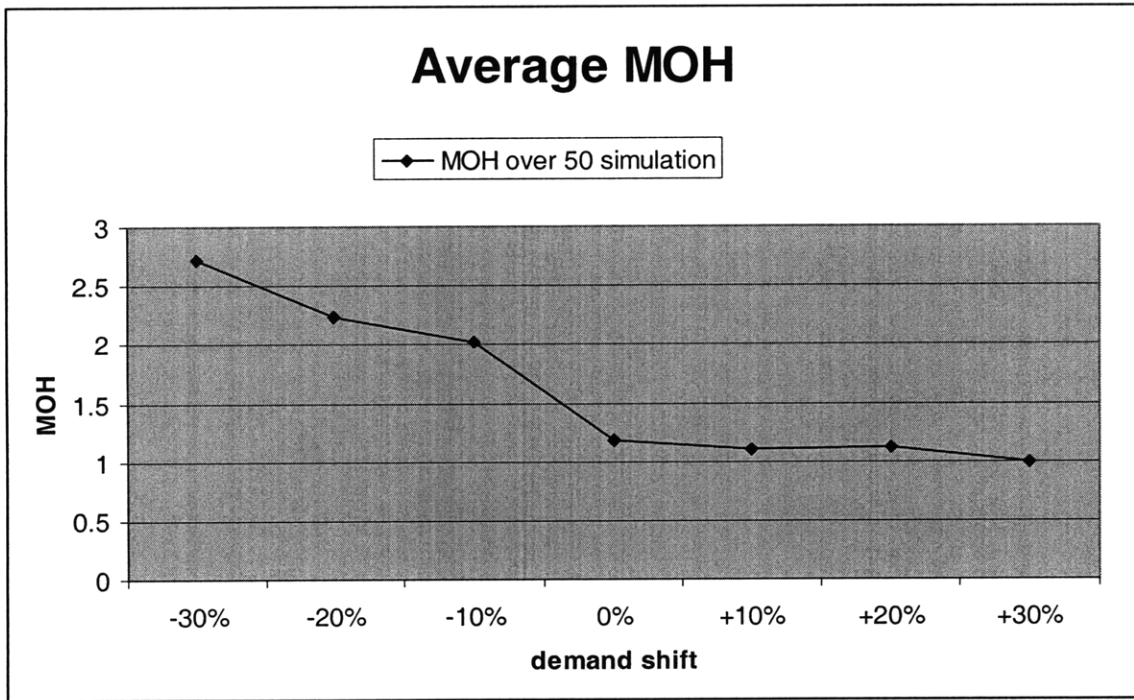
**Figure 10-5** Comparison between theoretical and simulated loss for different solutions of the proposed policy

As it can be seen in figure 10-5, the optimization and the simulation graphs show a similar behavior, supporting the correctness of the optimization model.

This curve led to the selection of a solution providing an average inventory level of 1.2 months on hand, as described in section 10.2.

### 10.3.2 Robustness analysis

By running the proposed inventory levels over statistical demand, the robustness of the proposed policy can be tested, as described in Serra [27]. The statistical demand is generated using the distribution of demand of each system and item over the previous two years. In the following example, the simulation is run 50 times for seven different values of shift in demand. The shift in demand, however, is not taken into account in calculating the proposed inventory levels. Figure 10-6 depicts the average inventory months on hand versus the shift in demand.



**Figure 10-6** Simulated average MOH vs. demand shift

As the demand decreases the proposed policy shows a steep increase in the MOH (above the limit of 2), while, when there is an increase in the volumes, the months on hand remain substantially stable but there is a considerable increase in the lost sales. This suggests the need for the inventory planner at Instron to update the control parameters as soon as a shift in the demand is detected, using the provided tools.

# 11. Recommendations

## 11.1 Introduction

As showed in section 10.2, the optimized control parameters result in a decrease of 35% in the inventory MOH. Moreover, it is estimated that extending the optimization to all the accessories in the Configuration Department would reduce the MOH by a similar percentage. Finally, as mentioned in section 10.1, the raw materials inventory policy provided would cut the parts inventory value on hand by 30% (or even 46% if shorter lead times are agreed with suppliers).

This represents a substantial motivation to extensively use the software provided, which allows computing the replenishment parameters for all the Instron accessories both at the finished goods and part levels, and integrate it into the Manufacturing Department procedures.

The following recommendations are made to the Instron workers in order to properly implement the proposed policy and allow improvements in the future:

- Compute the inventory levels for the raw parts using the proposed tool as frequently as possible
- Compute the inventory levels for the finished goods using the proposed tool as frequently as possible
- Keep the data on IBS updated as the accuracy of the solutions depend on the quality of available data
- Keep track of the lead times for both raw parts and finished goods
- Use the provided tool to evaluate the benefits of negotiating better lead times from the suppliers

## **11.2 Discussion**

### ***11.2.1 Updating inventory levels***

In order to guarantee that the optimal mix of accessories is on the shelves, the inventory planners of the Configuration Department should periodically update the proposed inventory control framework using the most recent sales records available. The computation of the control parameters can be performed with the provided software.

The rapid changes that can occur in the demand, in fact, dictate the need to update the replenishment quantities as frequently as possible. On the other hand, changing the parameters implies a cost in terms of time: the time required to gather the data, run the executable file and insert the new values in IBS. This might imply negotiating new quantities with the suppliers, when agreements exist. Since it is common practice at Instron to update the IBS records at the beginning of every quarter, there is the opportunity to combine these operations and perform the computation every quarter, in time for the data of last quarter to be fully available.

A further decision to be taken by the software operator concerns the quantity of sales data to include in the analysis, for the statistical characterization of the demand and the computation of the Virtual Profit. One year is the minimum time interval that should be considered to properly estimate the variations. As the considered time period increases, the computation time increases as well. Moreover, since there is continuous variation in the product list and in the market, including older data in the analysis implies greater differences between the historical data and the current situation.

In order to minimize the run time and achieve accurate results, the sales records of the last four quarters should be used. As an example, if the analysis is performed in July, the planner should collect the data for the third and fourth quarters of the previous year and for the first and second quarter of the current year.



### ***11.2.2 Shift in demand***

As mentioned in Chapter 1, historical sales are used to estimate the future demand. While it is reasonable to assume that the relationships among products (the correlation) and the variations in the demands resemble the ones of the previous year, shifts in the average volumes can occur from one year to another. When a forecast of the shift is available, it should be entered in the command shell of the software, which is able to take this factor into consideration and to provide control parameters that fit the actual situation.

### ***11.2.3 Dividing the analysis***

In order for the information involved to be easily managed, the control parameters should not be optimized for all the items at the same time. In fact, because IBS does not currently provide all the quantities needed for the analysis, a manual integration of data is required. For example, the operator has to manually enter lead times for the items considered when not available and check for the accuracy of other parameters, such as unit costs and lot sizes, when unexpected results are detected. Moreover, the optimization of the part level replenishment quantities involves downloading the bill of materials for all the considered products and the complexity of this operation increases with the number of products. Therefore the items should be divided into groups sized so that the operator is comfortable with their management.

The division of the analysis in groups of items allows focusing on the accuracy of the inputted data which is critical for the correct performance of provided software. As an example, the inaccuracy of the lead times data provided by IBS can lead to store inadequate quantity of items.

Similarly, even if the simulation would be a closer representation of the factory floor since more items will be simulated, the run time would become large and the results difficult to interpret.

#### **11.2.4 Lead times accuracy and negotiation**

As demonstrated by Palano [26], the correct estimation of the replenishment lead times could lead to a saving of 30% in terms of VOH.

This suggests the need to improve the recording criterion for this type of data, which is currently based on many criteria. For some items that are on Kanban and for the parts that come from Binghamton (another Instron facility) the values of the lead times are known. However, for the majority of the items the lead time corresponds to the maximum lead time that can be tolerated from the supplier. As the cost and the yearly volume of one item increase, the less quantity can be stored for that item and the less time the company can wait for the supply to arrive. Also regarding the finished goods levels, lead times are missing on IBS for the parts assembled or reworked in the Norwood facility. For these parts, in fact, while setup time and run time are usually available, the time that elapses between the arrival of the order and the moment the product is ready is not recorded. The latter, however, is necessary for the computation of the optimal inventory levels.

Sufficiently accurate values can be obtained by using a new recording procedure and integrating it into IBS. Whenever an order is placed to the supplier, the purchasing agent should register the date and the supplier code, assigning a unique code to this record. The same identification number should be used in the receiving area to register the arrival date as soon as the order gets to the Norwood facility. In this way, by comparing the records with the same identification numbers, it is possible to track the lead times for all the items and suppliers so that they can be used in the computation of the inventory control parameters. When variability is present, the statistical distributions of the lead times can be evaluated. The availability of this type of data would potentially allow an extension of the optimization tools which consider stochastic lead times, as described in section 12.1.

As also showed in the raw materials control, a more drastic drop in the VOH can be achieved by negotiating shorter lead times with the suppliers. Whenever negotiation is possible, the supply chain planners should use the provided tool to evaluate the possible

benefits of changing the lead times. In particular, they can compare the decrease in inventory value on hand with the eventual increase in purchasing cost.

### ***11.2.5 Product categories***

The category of a finished good (face, grip, fixture, etc.) is not stored by the IT system. However, as showed by Palano [26], the customer expectations differ for items belonging to different categories, and this record becomes important for the optimization tool. Right now such information can be found in the product catalog and in many other sources. However, keeping an updatable database or excel file with all the products divided by categories would help to easily identify this information and decrease the time necessary to gather the data needed for the finished goods optimization program.

### ***11.2.6 Warning messages***

For what concerns the information accuracy, the operator should take advantage of the warning messages displayed by the programs provided when unexpected results are detected. The user is provided with detailed instructions to follow when such events occur, and with the operating procedure for the calculation of the inventory control parameters. The detailed instructions are provided to the user with the software, and are not shown in this work.

### ***11.2.7 New products and substitutions***

Whenever new products are released and their replenishment quantities have to be calculated, the operator should provide a table containing information about the new items. Two cases can be considered:

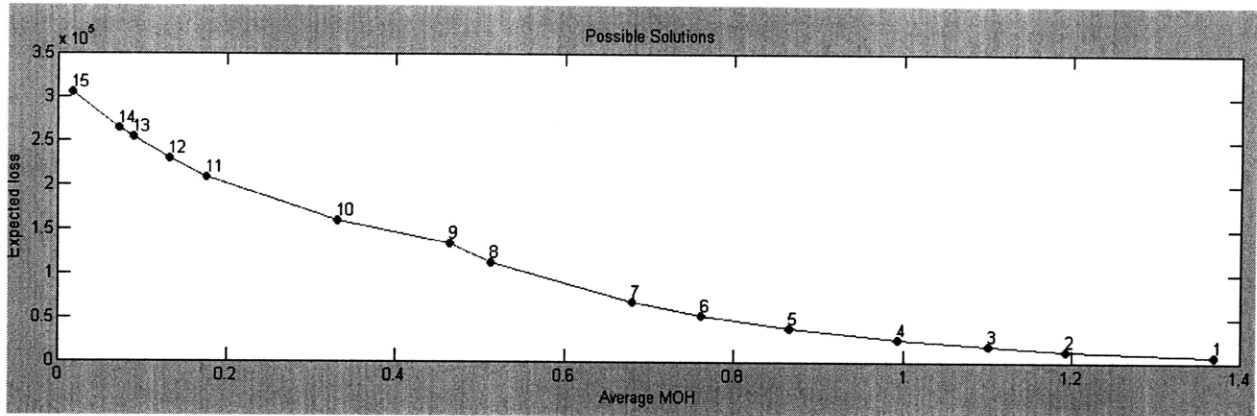
- If the new products directly substitute one or more items in the product list, those item should be indicated as well as the fraction of demand of the old product that would converge into the new one. This allows the program to estimate the Virtual

Profit and the statistical parameters of the new products demands based on the old sales data.

- If the new products are added to the product list and no old item is substituted, no historical sales data can be used to estimate the Virtual Profit, and the control parameters should be evaluated based on the simple Q,r model, without considering the correlation among the new items and the rest of the product list. In this case the operator is asked to provide a forecast of the future sales. This data is used to estimate mean value and standard deviation of the demand, and the z-factors are set by default to a high value which is not necessarily the optimal one, which cannot be estimated without knowing the Virtual Profit, but matches the need for the company to provide a high service level when the new items are introduced to the market.

### ***11.2.8 Selecting the best solution***

The final step of the computation of the control parameters involves the selection of the desired solution. Different solutions are provided, each one involving a different value of average MOH, and the operator is asked to choose one of them. A graph, similar to the one showed in figure 11-1, is displayed in order to aid the selection. For all the different solutions, the loss of sales profits and the MOH are plotted in the same graph and, as described in Facelli [27], the higher is the MOH, the smaller loss is achieved.



**Figure 11-1** Expected loss from sales vs. average inventory VOH for different solutions of the optimization.

When making this decision, the operator should consider that the displayed MOH is an average value and may fluctuate depending of the variability of the sales volumes. Because the Instron demand is subject to consistent fluctuations, the operator should not choose a value close to 2 MOH, which is the maximum value allowed at Instron. At the same time a small loss from sales should be achieved. This curve usually shows a flat tail, where for a little increase in the inventory cost only a little portion of sales is redeemed. The starting point of the flat tail can be considered a satisfactory solution.

### **11.2.9 Using and adjusting the recommended quantities**

The output of this computation is a list of recommended minimum quantities and reorder quantities, which are the parameters used to build the Kanban cards. While the reorder quantity coincides with  $Q$  in the  $Q,r$  model, the minimum quantity is  $R+1$ . The reason for this is that the minimum quantity indicates the number of items contained in a bag; when the bag is opened to take one part the level  $R$  is reached and the order is placed.

At this point, the operator has the chance to modify the proposed quantities if constraints are present. For instance, constraints on the lot sizes exist. In addition, some items have to be ordered or assembled in lots that are multiples of some predetermined quantity.

After the quantities are updated according to these constraints, a sensitivity analysis for the finished goods should be performed in order to evaluate the increase in the costs. The quantities can be directly modified in the Excel spreadsheet provided as output of the optimization tool, and the updated values of the theoretical MOH and VOH are showed. These quantities can be compared with the proposed ones and the choice must be taken accordingly.

A simulation can also be performed to observe the changes introduced by the adjusted quantities on the lost sales, value on hand and months on hand.

## **12. Future work**

As discussed in the Results and Discussion section, Instron has potential for improving its operations management. The result of this work is reducing wastes in the inventory management. Some topics from this research, which can be further explored, are:

### **12.1 Lead time variability**

Lead time variability is critical to every inventory policy. Variation in lead time can lead to unexpected stock outs or surges in inventory leading to increased costs and unsatisfied customers. This issue can be taken into account if the variation in lead time is known. If Instron Corporation keeps track of lead times as described in the recommendations section, the variability can be recorded and implemented inside the replenishment policy.

### **12.2 Manufacturing constraints**

Manufacturing constraints are essential on a factory floor since mostly limited work force is available to accomplish tasks. Orders sometimes need to be rescheduled, or in the worst case lost, if manufacturing constraints and pending commitments are not taken into consideration while promising a lead time to a customer. Thus, while determining the finished goods and part levels, it is important to consider the manufacturing constraints since if these are not considered, unrealistic levels will be obtained. At the same time, the initial analysis has revealed that most of the manufacturing constraints are both independent and difficult to quantify.

Currently, final finished good levels are checked and compared by the inventory planning team before implementing. Also, the lead times have been estimated considering the effects of capacity constraints. However, the optimum method to implement this would be to consider the constraints inside the optimization and simulation itself. This will make the new inventory levels faster to implement and easily reusable.

### **12.3 Include back orders in the simulation**

As discussed in Chugh [28], simulation has been developed on a simplified model of the manufacturing floor. Back orders have not been considered in the simulation and immediate order execution is being done. However, in reality, back orders will cause the orders to wait longer than required. Implementing back orders in the simulation is a complex process and needs the creation of a new database to keep track of them. Also, some orders are unexpectedly delayed due to incomplete payments, quality audits, etc. A more accurate picture can be obtained if back orders and manufacturing time is considered inside the simulation.

### **12.4 Include part level into the simulation**

Currently the simulation tool only considers the finished goods level. The part level inventory has been determined directly under the condition that it has to be available with a very high probability whenever the finished goods need to be prepared. This, however, is an approximation and there is a miniscule probability that an order cannot be satisfied if a part level inventory is not available. Thus, it is required that a simulation be built which starts from the part level inventory, develops finished goods and finally executes the orders. This simulation will be a more accurate representation of the factory floor.

### ***12.5 Q,r policy using Poisson distributed demand***

As shown in Serra [27], Instron's monthly demand for frames can be better approximated with a Poisson distribution [26]. The assumption of normally distributed and continuous demand fits well the reality if the average demand is large enough. However for many products at Instron the sales volume is limited and it might then be interesting to perform a similar analysis with a Q,r policy assuming Poisson distributed demand. An in depth



study can provide detailed results on whether changing the demand distribution can lead to increased profits.

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